

# Can Nuclear Power Supply Clean Energy in the Long Run?

by

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## Abstract

This paper models nuclear energy by developing a Hotelling model with endogenous substitution among polluting nonrenewable resources. We find that continued expansion of nuclear capacity at historical rates is likely to cause a scarcity of uranium and make nuclear power costlier than other energy sources. However, technical progress in the nuclear sector could mitigate resource constraints and supply carbon-free energy for an indefinite period. However, the role of nuclear power may be much diminished with higher rates of technological change in conventional and renewable energy resources. The price-induced substitution of energy resources in the model suggests that the shadow price of carbon may be an order of magnitude lower than previously estimated. A policy implication is that environmental targets could be met at a significantly lower cost than generally assumed.

*Keywords: Energy Resources, Environmental Regulation, Global Warming, Hotelling Theory,*

*Resource Substitution*

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## 1. Introduction

Nuclear power accounts for a sixth of all electricity production globally. Seventeen countries depend on it for at least a quarter of their electricity (World Nuclear Association, 2003). The United States has 103 plants that generate 20% of its electricity and France has 56 of them that account for 80% of electricity supply. Global nuclear generation capacity has exhibited double digit growth in recent years and continues to grow rapidly in the developing countries. About 36 new reactors are under construction. China which has 9 plants, expects to build 30 more in the next 15 years.

Even though the developed countries have not built any new nuclear plants for some time, there is a resurgence of interest in nuclear power as a clean alternative to polluting fossil fuels.<sup>2</sup> The ratification of the Kyoto Protocol into a binding international treaty has revived interest in non-carbon energy alternatives including nuclear energy. Limiting the use of carbon-emitting fossil fuels such as coal, oil and natural gas which currently account for nearly 85% of global energy consumption will mean increased use of nuclear energy, since hydro and renewable energy sources can not supply large volumes of baseload power. In the U.S., nuclear power has been used to replace coal to meet standards set by the Clean Air Act, especially in the northeast region.<sup>3</sup>

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<sup>2</sup> Some environmentalists including a founder of the group Greenpeace have voiced support for a pro-nuclear energy strategy (Washington Post, 2006).

<sup>3</sup> "Most of the avoided carbon dioxide emissions over the last 20 years have come from nuclear power," according to a U.S. Department of Energy official (Moniz, 1999).

This paper develops a long run model of energy substitution to examine the role of nuclear power as a source of clean energy supply. The economic modelling of nuclear power presents several methodological challenges. Since major energy resources are nonrenewable, we use a Hotelling framework in which prices reflect the scarcity of the resource. Nuclear power is strictly neither a nonrenewable nor a renewable resource. Its major input uranium is nonrenewable, but the output (reprocessed uranium and plutonium) can be re-used as input. This recycling of materials must be an important feature of any model of nuclear energy. We consider business-as-usual scenarios as well as technical change both in the nuclear industry and in conventional and renewable energy sectors.

There are relatively few studies of the long-run economics of nuclear energy. Cropper (1980) has examined a theoretical model of the trade-offs between fossil fuels and nuclear energy. Most energy models tend to assume the availability of nuclear energy at given prices, but do not account for the uranium used, which turns out to be a critical issue, as we see in this paper. Nordhaus (1979) pioneered this endogenous substitution approach to examine the impact of OPEC-induced oil price shocks on the U.S. economy and subsequently Chakravorty *et al.* (1997) modeled cost reductions in solar energy due to exogenous research and development.

Available supplies of uranium can be used only for a few decades before scarcity induced prices will make alternatives cheaper. Adoption of nuclear power is sensitive to assumptions about technical change. Technical progress in the nuclear industry in the form of adoption of modern Fast Breeder technology will lead to increased recycling of nuclear waste, and a steady state with nuclear power as the main source of energy. However, if the cost of fossil fuels and solar power

also decline due to technical progress, the role of nuclear energy is much diminished. The scarcity rent of uranium decreases significantly. This sensitivity to technical change is preserved when we vary energy demand and cost parameters, double initial uranium reserves and assume a faster expansion of nuclear generation capacity.

We find that substitution among energy resources leads to a significantly lower estimate of the shadow price of carbon. Most estimates in the literature suggest a range of \$100 to \$300/ton (in present value terms) of carbon and higher (see Fischer and Morgenstern, 2005; Gerlagh 2006). In our model, this price appears to be quite sensitive to assumptions regarding long-run technical progress. With conventional nuclear power, the price of carbon is \$40/ton, which decreases to \$10/ton with modest reductions in the cost of solar energy and fossil fuels. Both these figures are an order of magnitude lower than in other studies suggesting that models that do not account for price-induced substitution may overestimate the potential cost of environmental regulation.

Section 2 introduces a Hotelling model with environmental regulation. Section 3 summarizes the main elements of the empirical model with details and data provided in the Appendices. Section 4 discusses the simulation results. Section 5 concludes the paper.

## **2. A Dynamic Model with a Cap on the Stock of Emissions**

In this section we discuss the Hotelling theory with environmental regulation imposed in the form of a ceiling on the stock of pollution. Such a ceiling may be thought of as a target carbon concentration in the atmosphere (e.g., 550 parts per million). We assume one demand, one

polluting nonrenewable resource and a "clean" backstop resource.<sup>4</sup> The main conclusion here is that because of this constraint on the stock of emissions, we may observe the joint use of the two resources before a complete transition to the latter.

Let the instantaneous utility at time  $t$  generated by energy consumption  $q(t)$  be given by  $u(q(t))$  which is assumed to be strictly increasing and concave in  $q$ , i.e.,  $u'(q) > 0$ ,  $u''(q) < 0$ . Both resources are assumed to be perfect substitutes. The consumption rates of the polluting and clean resource are given respectively by  $x(t)$  and  $y(t)$  respectively. Utility from energy is then given by  $u(x(t) + y(t))$ .

The initial reserves of the nonrenewable are assumed known, denoted by  $X(0)$ . Let its unit extraction cost be  $c_e$ . Define the pollution generated per unit of this resource be  $\theta$ . Then  $X(t)$  is the residual resource available at time  $t$ , where  $\dot{X}(t) = -x(t)$ . Let  $x_c$  be the consumption for which the marginal utility is equal to the unit cost, i.e.,  $u'(x_c) = c_e$ . Denote  $Z(t)$  to be the stock of pollution at time  $t$ , with  $Z(0)$  as the initial stock. Pollution increases  $Z(t)$ , but a portion declines naturally at an assumed rate  $\alpha > 0$ . That is, the growth of the pollution stock is given by

$\dot{Z}(t) = \theta x(t) - \alpha Z(t)$ . Define the exogenous ceiling on the stock of pollution to be

$\bar{Z}$  with  $Z(0) < \bar{Z}$ . Then we can define  $\bar{x}$  as the maximum consumption rate of the nonrenewable resource if  $Z(t)$  equals its ceiling  $\bar{Z}$ , i.e.,  $\bar{x} = \alpha \bar{Z} / \theta$ , and by  $\bar{p}_e$  the corresponding marginal utility, so that  $\bar{p}_e = u'(\bar{x})$ .

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<sup>4</sup> The purpose of this model is to develop insights on how a ceiling affects the extraction of the non-renewable resource.

Finally, let  $c_r$  be the constant unit cost of the abundant backstop resource which is assumed higher than the cost of the nonrenewable,  $c_r > c_e$ . Let  $y_c$  (analogous to  $x_c$ ) be the extraction rate for which the marginal utility equals the unit cost of the backstop, i.e.,  $u'(y_c) = c_r$ . The social planner chooses extraction rates of the two resources to maximize welfare as follows:

$$\max_{\{x(t), y(t)\}} \int_0^{\infty} \{u(x(t) + y(t)) - c_e x(t) - c_r y(t)\} e^{-\rho t} dt$$

subject to the two differential equations  $\dot{X}(t)$  and  $\dot{Z}(t)$ , and given values of  $X(0), Z(0)$  and  $\bar{Z}$ .

The current value Lagrangian is

$$L(t) = u(x(t) + y(t)) - c_e x(t) - c_r y(t) - \lambda(t)x(t) + \mu(t)[\theta x(t) - \alpha Z(t)] \quad (1)$$

The first order conditions are

$$u'(x(t) + y(t)) - c_e - \lambda(t) + \theta\mu(t) \geq 0 \quad (= \text{if } x(t) > 0), \text{ and} \quad (2)$$

$$u'(x(t) + y(t)) - c_r \geq 0 \quad (= \text{if } y(t) > 0). \quad (3)$$

The dynamics of the system is determined by

$$\dot{\lambda}(t) = \rho\lambda(t) \Rightarrow \lambda(t) = \lambda(0)e^{\rho t}, \text{ and} \quad (4)$$

$$\dot{\mu}(t) = (\rho + \alpha)\mu(t) + v(t), \quad v(t) \geq 0, \quad (5)$$

with  $v(t)[\bar{Z} - Z(t)] = 0$ . Here  $\lambda(t)$  is the usual Hotelling rent and  $\mu(t)$ , which is non-positive, is the shadow price of a unit of pollution stock. Lastly, the transversality conditions are

$$\lim_{t \uparrow +\infty} e^{-\rho t} \lambda(t) X(t) = \lambda(0) \lim_{t \uparrow +\infty} X(t) = 0, \text{ and } \lim_{t \uparrow +\infty} e^{-\rho t} \mu(t) Z(t) = 0.$$

The necessary conditions can be interpreted easily. Condition (2) equates the marginal benefit of an additional unit of the non-renewable to its total marginal cost, which includes the unit cost of extraction  $c_e$ , the scarcity rent  $\lambda(t)$ , and the externality cost  $-\theta\mu(t)$ . Equation (3) equates the marginal benefit from using the backstop to its unit extraction cost  $c_r$  and (4) suggests that  $\lambda(t)$  must rise at the rate of interest (Hotelling, 1931). Shadow price  $\mu(t)$  must increase at a rate equal to the sum of the discount rate and the natural decay rate of pollution except when the ceiling is binding, in which case the value of the constraint  $v(t)$  is non-zero. Finally at the end of the planning horizon, the value of the resource and pollution stocks must also go to zero.

One solution to the above problem which is consistent with the results of the models run later in the paper is shown in Fig. 1. The curve  $MC_A$  represents the unit extraction cost plus the shadow price of the non-renewable over time absent environmental regulation. This is the Hotelling model with no pollution. The polluting resource is consumed from the beginning until time  $T$ , when it is exhausted and the backstop is used at rate  $y_c$ . The curve  $MC_B$  represents the marginal cost of the non-renewable with the ceiling constraint, and includes its extraction cost and the shadow price plus the shadow price of pollution.  $MC_B$  increases to equal the cost of the backstop  $c_r$  at time  $t_1$ . At this time, the stock of pollution also reaches the ceiling. However, at price  $c_r$ , demand is too high to be satisfied only by the nonrenewable without violating the ceiling, hence

some backstop is used. From  $t_1$  to  $t_2$  the pollution level is at its maximum. The extraction rate of the non-renewable is the maximal rate  $\bar{x}$ , and the marginal costs  $MC_B$  and  $c_r$  are equal. The addition to the stock of pollution equals the natural decay,  $\theta\bar{x} = \alpha\bar{Z}$ . The non-renewable gets exhausted at  $t_2$  and the backstop supplies all energy. The ceiling is not binding from time  $t_2$ , and the stock of pollution declines gradually to zero. Beginning from  $t_2$  the shadow price of pollution is zero, and  $MC_B$  is higher than  $c_r$ . Hotelling scarcity rents in the model with regulation, depicted by  $MC_C$ , are naturally lower than in the unregulated case,  $MC_A$ .

Equilibrium quantities are shown in the lower graph of Fig. 1. The dashed curve corresponds to the pure Hotelling path without regulation. Resource extraction declines to  $y_c$  at time  $\Gamma$ , followed by use of the backstop. The solid lines show resource use under regulation. Note that  $t_1 < \Gamma < t_2$ . Regulation initially slows down the extraction rate of the non-renewable until  $\Gamma$ , but extends the time period during which it is used, since cumulative demand in both cases must equal the initial stock. If the maximal pollution rate  $\bar{x}$  is larger than  $y_c$ , only the non-renewable will be extracted at the ceiling followed by a Hotelling-type transition to the backstop resource. Other patterns may arise depending upon the relative magnitudes of  $\bar{x}$  and  $y_c$ .<sup>5</sup>

The main point of the above model is to show that when a ceiling is imposed on the stock of pollution, extraction may increase for a time, then stay at the ceiling when both the fossil fuel and the clean resource is used simultaneously until the exhaustion of the former.

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<sup>5</sup> See Chakravorty, Magne and Moreaux (2006) for a complete analytical solution in a more general framework.

### 3. The Simulation Model with Fossil Fuels and Nuclear Power

We apply the framework outlined above but with several demands and nonrenewable resources, nuclear technology with recycling of materials and a backstop renewable resource (solar energy).

We focus on the main economic features of the model and provide details of the nuclear technology and data in Appendices A and B. There are four sectors in this energy economy - specific electricity, transportation, residential and commercial, and industrial. They are characterized by independent demands that are a function of energy prices and income.

Generalized Cobb-Douglas demand functions for each sector are given as  $D_j = A_j P_j^{\alpha_j} Y^{\beta_j}$ , where  $\alpha_j$  and  $\beta_j$  are respectively the price and income elasticities for demand in sector  $j$ ,  $A_j$  is the sector-specific technical coefficient,  $P_j$  is the price of delivered energy in sector  $j$ , and  $Y$  is global GDP which is non-stationary. GDP increases exogenously over time at a declining rate.

We consider the three major fossil fuels - oil, natural gas and coal. We approximate their reserves with different extraction costs by considering two grades each of oil and natural gas (as in Nakicenovic and Riahi, 2001), and since coal is much more abundant, three grades of coal, each with constant extraction cost. Conversion costs for each resource into each demand are added to these extraction costs.

Nuclear technology is optimized by choosing the amount of energy produced by conventional Light Water Reactor (LWR) technology. Technical progress in the nuclear sector is modeled by assuming that Fast Breeder Reactor (FBR) technology is available.<sup>6</sup> The model of nuclear power is embedded in the general model of substitution across resources and demands. The stocks of oil,

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<sup>6</sup> The LWR is the standard nuclear technology most commonly used. It uses uranium and produces a significant volume of waste. The FBR is a modern technology with higher capital costs, prototypes of which are operational. It uses uranium and plutonium and recycles a larger portion of the waste. See Appendix for further details.

coal, natural gas and uranium are finite. Thus each of them has a Hotelling scarcity rent. The backstop resource, solar energy can produce electricity at constant unit cost. The combustion of fossil fuels releases carbon into the atmosphere. Nuclear power is carbon free. LWR technology uses uranium ore as input. FBR technology uses a mix of several inputs, including wastes from LWR production. The algorithm chooses the least cost energy supply for each sector. The two nuclear technologies enjoy complementarities in materials use and waste recycling and thus may be deployed jointly. Unlike for fossil fuels, production of nuclear energy creates the need for reprocessing and storage of wastes. Their disposal is costly and must be included in the total marginal cost of nuclear energy. In models where only LWR technology is available, these wastes do not have economic value so their shadow price is zero. However, when FBR technology is an option, wastes have economic value as inputs in FBR operation, so they have a non-zero scarcity rent. Consumer plus producer surplus is maximized subject to the technological relationships and stock dynamics. We consider several scenarios, described as follows:

A. The Baseline Model: Fossil Fuels with No Regulation: This model is run with known estimated stocks of fossil fuels - oil, coal and natural gas, and solar energy as the backstop. It has no nuclear power and no cap on carbon emissions.<sup>7</sup>

B. The Business-As-Usual (BAU) Model: Regulation with Standard Nuclear Technology  
This scenario imposes a clean carbon target with conservative assumptions on the growth of nuclear power. We only consider LWR technology. This may represent a policy environment in which there are no major technological improvements in nuclear technology. It may be unrealistic to expect an instantaneous expansion of nuclear capacity given the long lead times (about 7-10

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<sup>7</sup> This is the Chakravorty *et al.* (1997) model with updated energy data. We have ignored the contribution of existing nuclear capacity as well as other resources such as hydroelectricity and biomass, all of which together account for about 15% of current supply.

years) involved in licensing, constructing and commissioning of nuclear power plants. So we impose an upper bound on the growth rate of nuclear capacity equal to a 1.6% annual increase (OECD, 2000a).<sup>8</sup> The cap on the stock of emissions is set at 550 ppm.<sup>9</sup>

*C. The Pro-Nuclear Model: Regulation with Technical Progress in Nuclear Energy:* Nuclear production capacity is assumed to increase faster when both nuclear technologies are available. When FBR technology is an option, we impose a maximum 1.6% annual rate of increase (same as in case B) in nuclear generation until 2020 then double this expansion rate for future periods. This model captures a pro-nuclear policy environment, with availability of FBR technology and a faster growth in generation capacity in the future.

*D. The Pro-Nuclear Model plus All-round Technical Progress in Fossil Fuels and Solar Energy:* Here technical change is modelled through exogenous reductions in costs. Since investment and operating costs are significant in the nuclear industry - ranging from 60 to 80%, depending on the discount rate used for calculations - we apply a cost reduction of 4.4% per decade, the rate chosen such that the unit cost of FBR technology decreases to the level of LWR technology in about 50 years.<sup>10</sup> The cost of LWR technology is assumed constant. It may be that this technology has achieved maturity and future learning-by-doing effects may be relatively small. Potential cost reductions may be compensated by increased safety and proliferation measures which must be part of any major nuclear expansion program (MIT, 2003). The cost of electricity from solar energy is assumed to decrease at a rate of 20% per decade.<sup>11</sup> We assume that fossil fuel

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<sup>8</sup> This is a conservative estimate. Nuclear energy production has grown by a factor of 12 between 1973 and 2000, which is equivalent to an annual average increase of about 12% (IEA, 2001). We also run a model with no capacity limits on nuclear expansion (see below) but the results did not change significantly.

<sup>9</sup> A 550 ppm carbon cap is applied to all the models, except model F, which examines sensitivity to alternative carbon targets.

<sup>10</sup> It is hard to predict the learning by doing and other benefits of FBR adoption since the technology is still at the prototype stage. However, they are expected to be significant, as suggested by several studies (e.g., MIT, 2003). We apply a modest learning rate.

<sup>11</sup> This figure approximates the slope of the learning curve for photovoltaic electricity (OECD, 2000b).

conversion costs are reduced by 1% every five years, which leads to approximately constant end-user energy prices, with cost reductions offsetting the increase in scarcity rent.<sup>12</sup> Extraction costs of fossil fuels are maintained at previous levels.

*E. The Pro-Nuclear Model with Stagnation in the Demand for Energy:* While the previous cases examined the near term scenario of increasing demand, recent empirical evidence suggests that fertility rates and population have begun to decline in many countries that have historically exhibited high rates of growth (Lutz, Sanderson and Scherbov, 2001).<sup>13</sup> With the aging of these societies, energy demand will likely follow this declining trend, albeit with a time lag that accounts for the expected increases in energy consumption per capita for residents of developing countries. We use these population and GDP per capita assumptions to consider the long-run case when demand for energy stabilizes with time. We model an increase in the GDP growth rate in the near term followed by a decline, which reflects more accurately the expected fall in population and levelling of per capita energy use in the long run. In all other scenarios analyzed, we assume a steady decrease in GDP growth over time (see Appendix B). In this run, both nuclear technologies are available and there is no technical progress.

*F, G. Sensitivity Analysis with Alternative Emission Caps and Growth of Nuclear Capacity:* We examine the sensitivity of resource use, emissions and the shadow price of carbon to alternate carbon targets in the range 450-650 ppm (model F) and remove capacity constraints on the expansion of LWR nuclear power (model G).

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<sup>12</sup> In general, fuel prices have been constant because of technical progress (Parry, 1997). Ignoring market incentives for fossil fuel cost reduction may result in underestimating extraction and emission rates. An important advantage of the assumption of approximately constant resource prices is that it responds to critics of the Hotelling theory who argue that historically, long-run resource prices have not exhibited scarcity effects and models that rely on Hotelling-induced resource price movements may be mis-specified.

<sup>13</sup> According to Lutz *et al.*, world population is expected to rise from its present level of 6 billion to about 9 billion in 2070, then drop to 8.4 billion in 2100. Fertility rates have fallen below replacement levels in many developed and developing countries. Other UN predictions suggest that the population peak may occur sooner, reaching 7.4 billion in 2045 before starting a decline (United Nations, 2002).

#### 4. Model Results

Table 1 shows the time profile of energy substitution in the baseline case (model A) with time intervals in decades. Coal is used in electricity generation, oil in transportation and natural gas in residential and industrial sectors. Both oil and gas get exhausted ultimately and are replaced by coal. Coal reserves are much larger than that of the other two fossil fuels, therefore its scarcity rent is low. Finally all sectors transition to the backstop resource, solar energy. Table 2 shows the share of each fuel by sector at various dates. Natural gas supplies the bulk of energy consumption during the next 50 years and then declines while the share of coal increases to make up the deficit.<sup>14</sup> The general shift from coal to natural gas in electricity generation in recent years supports this conclusion. The heavy dependence on coal in this nuclear free scenario also raises carbon emissions. The carbon concentration (see Fig. 2) reaches a level of 750 ppm in the year 2100 and peaks at 1,620 ppm in 2260, which is generally believed to be in the range that may cause significant damages (Alley *et al.*, 2003). Emissions reach a maximum of 43 billion tons in 2180.<sup>15</sup>

We next impose pollution regulation and consider the use of nuclear energy. Table 3(a) shows resource use when only LWR technology is available (model B), and Table 3(b) shows both technologies (LWR and FBR, model C). Consider only the LWR option first. A cap on the stock of emissions raises the cost of fuels emitting carbon. The costs of coal, oil and gas rise in decreasing order due to their respective pollution contents. Nuclear energy becomes immediately

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<sup>14</sup> The model underestimates the share of oil in the initial years and overestimates the use of natural gas. Our price elasticity estimates for transportation (taken from Nordhaus (1979) and Chakravorty (1997) are somewhat higher than what they should be. However, this does not affect resource specialization. The critical issue of interest is substitution of nuclear energy for coal in electricity, which is not affected.

<sup>15</sup> This is several times more than current annual emissions of about 7.35 billion tons.

economical in electricity generation and is used at maximum capacity, jointly with coal until uranium is exhausted (by 2100). This solution is as described in Fig.1. Solar becomes competitive upon depletion of uranium, because the high shadow price of carbon makes coal expensive. Compared to the model without regulation, the cost of coal in electricity increases from 5 to 6.5 cents per kWh for the base year 2000.<sup>16</sup> Coal is used in electricity when both oil and gas resources run out, except in transportation where solar has an advantage relative to coal, as shown in the table. Oil and gas retain their comparative advantage in the other sectors. The carbon ceiling is reached in about 100 years (see Fig. 2). The scarcity rent of uranium is a high \$1,542/kg (see Table 5).<sup>17</sup> Uranium is exhausted within this century, even with the phased expansion of nuclear capacity in the model. Coal and solar power are used when oil and gas are depleted.<sup>18</sup>

With the availability of FBRs in model C, it becomes economical in electricity generation in the year 2075 (see Table 3(b)). Because of the higher fuel efficiency of FBR, nuclear power ultimately supplies energy in all sectors and its share rises over time (see Table 2). Some coal is used jointly with nuclear in the transportation sector once oil is exhausted. Only 16% of aggregate coal is used in this scenario, and the scarcity rent for coal is zero. The available reserves of oil and natural gas however, continue to be exhausted and their respective comparative advantage in transportation and the residential and industrial sectors are unaffected

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<sup>16</sup> With a carbon cap but without nuclear power, the high price of carbon prevents coal from being used for electricity production in the initial period. The electricity sector is supplied by oil for the next 30 years. Solar becomes competitive early in this sector, in year 2090. These results are not shown in the paper.

<sup>17</sup> Some studies (e.g., MIT, 2003) have suggested that the price of uranium is a small fraction of the total cost of nuclear power. Our analysis, which treats uranium as a finite resource, indicates that even if the cost of uranium is small, its scarcity rent is likely to be quite high. A high shadow price of uranium, however, will trigger new discoveries of the resource, and augmentation of the initial stock assumed fixed in our case. With these augmented reserves, the shadow price of uranium is likely to be lower than what is reported here. See Table 5 for results with a doubling of the uranium reserve.

<sup>18</sup> In our model, nuclear capacity is endogenously determined, but these estimates are in the range of engineering feasibility studies which suggest that current reserves of uranium will support expansion of nuclear power for a period of 40 years (see MIT, 2003).

by the availability of modern nuclear technology.

With both nuclear options, LWR technology is abandoned around the middle of this century and FBR supplies all of the nuclear energy (Table 3(b)). When oil and gas reserves get exhausted, nuclear power supplies electricity which is then converted for use in the residential, industrial and transportation sectors. Coal is still used because it is the cheapest among all energy resources. However, because the carbon concentration is at its maximum allowable limit, the quantity of coal that can be used is limited to the amount that compensates for the atmospheric removal of carbon as discussed in the theoretical section of this paper. With modern nuclear technology, solar power is no longer used. The available supply of uranium is enough to generate the plutonium needed to run the FBR facilities. These results represent the "best case" for the expansion of nuclear power.

When costs decline exogenously (model D), nuclear power becomes less important. Both nuclear technologies are used, but solar energy substitutes for nuclear power late this century (see Table 4). Electricity from solar and nuclear energy is used in the transportation sector. The residential sector continues to use cheap gas, which along with oil also provides energy for the industrial sector. Exogenous cost reductions make coal relatively costly and about 2% of it is used in the aggregate. Lower grades of oil become cheaper with time and are stretched out for a relatively long time. Cost reductions lead to a decline in the shadow price of carbon to \$10/ton of carbon in the year 2000 (see Table 4). The carbon concentration reaches a ceiling at the beginning of the next century, but stays at that level only briefly (see Fig. 2). This suggests that modest technical progress in other clean fuels (such as solar energy) may reduce the role of nuclear energy and the time during which the ceiling is binding (compare the bottom two graphs in Fig. 2).

The present value (in year 2000) of the scarcity rent of uranium and the imputed price of a ton of carbon is shown in Table 5. Uranium prices are generally high except in the case with technical change (\$17/kg). They are at their maximal levels when only LWR technology is an option, and remain significant even with a doubling of the stock of uranium. They decline significantly when both technologies are available, because of the complementarities in the recycling of wastes. The model with standard nuclear technology is driven mainly by coal, since uranium is in short supply. The intense use of coal drives up the price of carbon to \$40/ton (see Table 5). The price of carbon decreases with an increase in the target carbon level (model F) and is highly sensitive to it, as seen in the table. It decreases with availability of FBR technology because coal is abandoned much earlier in this case.

With stagnation in energy demand (model E), less carbon emitting fuels are used, which decreases the shadow price of carbon (Table 5) from \$26 to \$22/ton. However, this decline also makes carbon emitting fuels cheaper, and leads to a small increase in energy consumption and emissions. The scarcity rent of uranium falls from \$160 to \$95/ton. About 6% of the aggregate coal stock is ultimately extracted. Stagnation in energy demand leads to a 35% reduction in cumulative energy use over the entire time horizon.

There is a tradeoff between the production of carbon and toxic wastes. Because of reprocessing, waste production is significantly lower under FBR technology than under LWR technology (Fig.3) despite more nuclear electricity generation.

A complete removal of the capacity restriction on nuclear power (model G) leads to a limited increase in the adoption of LWR technology. However, it does not affect the price of carbon (constant at \$41/ton, see Table 5) because there is a limited stock of uranium, and a faster expansion of nuclear power will mean a consequent decline in nuclear use later in time. Aggregate net benefits increase but not significantly. However, the role of nuclear capacity becomes more important when the target is 450 ppm. In this case, the price of carbon declines from \$207 (Table 5) to \$76 per ton.<sup>19</sup>

## 5. Concluding Remarks

This paper applies a generalized Hotelling model with price-induced substitution across resources to examine the role of nuclear power in reducing global warming. We find that nuclear energy may provide a clean alternative to fossil fuels, but only for a few decades. Given estimated stocks of uranium, the continued adoption of nuclear power at current rates could only be sustained temporarily. Even though the contribution of uranium to total nuclear costs is small, the scarcity rent of uranium is large and may cause nuclear power to become costlier than alternative fuels in the future. If expansion of nuclear capacity occurs at historical rates, uranium producers could engage in cartel-like behavior since the metal is found only in four countries, fewer than for crude oil.

The model which allows for price-induced substitution of energy resources also yields estimates of the shadow price of carbon that are an order of magnitude lower than those cited in the literature. Most studies do not allow for rising scarcity rents and fuel substitution, which may lead to model rigidities that overestimate the shadow cost of the externality. The shadow price of

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<sup>19</sup> The detailed results for model G are not shown but are available from the authors.

carbon plays an important part in determining which abatement options may be feasible as well as the size of a global permit market. Lower carbon prices may suggest that such a market may be smaller than expected, with lower benefits relative to no trading. The damage to economies that may be potential buyers of carbon, such as the United States, may be smaller than currently estimated. Similarly, potential benefits to sellers of permits such as Russia and Ukraine may be correspondingly smaller.

There are several restrictive assumptions in the model which could be relaxed in future work. The complexity of an empirical model with multiple resources and environmental regulation suggests that the textbook one-resource, one-demand Hotelling model may be inadequate for making a realistic assessment of energy alternatives. More work needs to be done in developing realistic theoretical extensions of Hotelling to consider important issues such as the recycling of resources. We have also abstracted from considering adjustment costs. Adding nuclear capacity in the form of a new plant or additions to an existing facility takes several years because of licensing and safety permitting procedures. We have assumed frictionless additions to capacity. To some extent, imposing a cap on capacity expansion in the model as we have done is a simple way of recognizing the existence of adjustment costs. However, the demand for clean energy far outstrips the potential supply through traditional nuclear power. Thus it is possible that adjustment costs may not make a big difference to the empirical results especially in the long run, although that needs to be checked. Adjustment costs will delay energy transitions between sectors and favor sectors with low adjustment costs such as fossil fuels and solar energy.

Although it is beyond the scope of this paper, a discussion of nuclear power is incomplete without

raising the issue of adverse public opinion, especially in the Western countries.<sup>20</sup> The objections are primarily three fold - waste disposal, reactor meltdowns and the possibility of weapons proliferation. FBR technology may partly mitigate the first concern by recycling a much higher proportion of waste by-products, although they also increase risk by requiring more handling. Loss-of-coolant accidents as in Three Mile Island and Chernobyl can be avoided in the future through "passively safe" reactors that are cooled by helium and lined with graphite pebbles (Hoffert, 2002). Various studies have suggested that the level of exposure for the present generation (over the next 100 years) due to direct radioactive releases from nuclear facilities into the environment is low and that the individual risk can be considered as "negligible" (Nordhaus, 1997). Others (cited by Radetzki, 2000) point out that the external costs of coal (from global warming and air pollution) are much higher than for nuclear - waste treatment and disposal and the low probability event of a catastrophe. However, public perceptions are different, and nuclear power is assumed to be more damaging than coal, in part because of the difficulty of comprehending low probability events (Viscusi, 1992). If anything these concerns suggest the need for more sophisticated economic models that incorporate differential risk between energy technologies.

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<sup>20</sup> The opposition seems limited to the building of new plants. For example in the United States, most of the 103 plants are in the middle of a licensing process (for another 20 years) that allows plants to seek permits for expansion of generation capacity.

## References

- Alley, R.B. *et al.* (2003), Abrupt Climate Change, *Science* 299 (March), 2005-10.
- Brown, C.E. (2002). *World Energy Resources*. Berlin: Springer-Verlag.
- Chakravorty, U., B. Magné and M. Moreaux (2006). A Hotelling Model with a Ceiling on the Stock of Pollution, in press, *Journal of Economic Dynamics and Control*.
- Chakravorty, U., J. Roumasset and K-P. Tse (1997), Endogenous Substitution among Energy Resources and Global Warming, *Journal of Political Economy*, 105, 1201-34.
- Charpin, J.M. (2000). Mission d'évaluation économique de la filière nucléaire. Le parc nucléaire actuel & La prospective technologique de la filière nucléaire. Commissariat Général au Plan.
- Cropper M.L. (1980). Pollution aspects of nuclear energy use. *Journal of Environmental Economics and Management*, 7, 334-52.
- Fischer, C. and R.D. Morgenstern (2005). "Carbon Abatement Costs: Why the Wide Range of Estimates?" Discussion Paper 03-42, Washington DC: Resources for the Future.
- Gerlagh, R. (2006). "A Climate-Change Policy Induced Shift from Innovations in Carbon-Energy Production to Carbon-Energy Savings", Unpublished Manuscript.
- Greene, R. (1999). *An Assessment of Energy and Environmental Issues Related to the Use of Gas-to-Liquid Fuels in Transportation*, ORNL-258, Oak Ridge National Laboratory, Oak Ridge, TM.
- Hoffert Martin, *et al.* (2002), Advanced Technology Paths to Global Climate Stability: Energy for a Greenhouse Planet, *Science* 298 (November), 981-87.
- Hore-Lacy, I. (2003), *Nuclear Electricity*, Seventh Edition. Uranium Information Center Ltd. and World Nuclear Association.
- Hotelling, H. (1931), The Economics of Exhaustible Resources, *Journal of Political Economy*, 39, 137-75.
- IEA (International Energy Agency) (2000). *Oil Prices and Taxes in the Year 2000, An IEA Statistical Fact Sheet*. International Energy Agency (IEA-OECD), Paris, France.
- IEA (2001). *World Energy Outlook 2001*. International Energy Agency (IEA-OECD), Paris, France.
- Kamien, M. and N. Schwarz (1991). *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, Elsevier Science Publishing, New York.
- Lutz, W., W. Sanderson and S. Scherbov (2001). The End of World Population Growth, *Nature*, 412, 543-46.
- Manne, A., and R. Richels (2002). *The Impact of Learning-By-Doing on the Timing and Costs of CO<sub>2</sub> Abatement*, working paper, Stanford University, Stanford, California.
- Massachusetts Institute of Technology (2003). *The Future of Nuclear Power. An Interdisciplinary MIT*

Study. Available online at <http://web.mit.edu/nuclearpower/>.

Moniz, E. (1999), *Shaping the Nuclear Future*, Annual International Symposium, The Uranium Institute.

Nakicenovic, N., and K. Riahi (2001). *An Assessment of Technological Change across Selected Energy Scenarios*, A IIASA-World Energy Council publication.

NEA (Nuclear Energy Agency) (1994). *The Economics of the Nuclear Fuel Cycle*. NEA-OECD publication, Paris, France.

NEA (2001). *Nuclear Energy Data 2001*. NEA-OECD publication, Paris, France.

NEA (2002). *Accelerator-driven Systems (ADS) and Fast Reactors (FR) in Advanced Nuclear Fuel Cycles: A Comparative Study*. NEA-OECD publication, Paris, France.

Nordhaus, W.D. (1979). *The Efficient Use of Energy Resources*, New Haven: Yale University Press.

Nordhaus, W.D. (1997). *The Swedish Nuclear Dilemma*, Resources For the Future, Washington, DC.

Nordhaus, W.D., and J. Boyer (2000). *Warming the World: Economic Models of Global Warming*, MIT Press, Cambridge, MA.

OECD (2000a). *Uranium 1999, Resources, Production and Demand*. NEA-OECD and IAEA publication, Paris, France.

OECD (2000b). *Experience Curves for Energy Technology Policy*. OECD publication, Paris, France.

Parry, I.W.H. (1997). Productivity trends in natural resource industries, *Resources For the Future Discussion Paper #97-39*, Washington, DC.

Radetzki, M. (2000). Coal or Nuclear in New Power Stations : The Political Economy of an Undesirable but Necessary Choice, *The Energy Journal*, 21, 135-147.

The New York Times (2003). *Will Calgary be the Next Kuwait?* by Manik Talwani, Aug. 14.

United Nations (2002). *World Population Prospects: The 2002 Revision, Highlights*. United Nations publications.

Viscusi, W.K. (1992). *Fatal Tradeoffs: Public and Private Responsibilities for Risk*, New York: Oxford University Press.

Washington Post (2006). *Going Nuclear: A Green Makes the Case* by Patrick Moore, April 16, page B01.

World Nuclear Association (2003), *Nuclear Power in the World Today*, Information and Issue Brief, June.

## Appendix A. The Nuclear Model with Recycling

### *Introduction*

Uranium is the main raw material used in the generation of nuclear power. Almost two thirds of the world's uranium reserves are found in four countries - Australia, Kazakhstan, Canada and South Africa.<sup>21</sup> In the Light Water Reactor (LWR), which is the most common technology used, mined uranium ore is enriched from 0.7% to 3.5%.<sup>22</sup> Uranium fissions to produce heat which is converted into steam that drives a turbine and produces electricity. The spent fuel contains most of the original uranium and some plutonium. This recovered uranium can be reprocessed, enriched and mixed with the plutonium in the spent fuel to produce a mixed oxide fuel that can be put into long term storage or reprocessed. We also consider a modern nuclear technology, the Fast Breeder Reactor (FBR), about 20 prototypes of which are in operation. These reactors are more efficient in using uranium. They use plutonium as base fuel but also produce it as waste. The FBR can extract approximately 60 times more energy from each ton of uranium than the conventional LWR. However, its higher capital costs and the present low price of uranium makes the FBR uneconomical.<sup>23</sup>

About 434 nuclear reactors are in service globally, representing an installed production capacity of 351 Gigawatts (GW). Another 36 reactors are currently under construction. Aggregate estimated reserves of uranium ore, including those already discovered and those recoverable at high cost, are estimated to be nearly 15.4 million tons (OECD, 2000a). We consider a single grade of uranium, but perform sensitivity analysis with a doubling of the stock.<sup>24</sup>

### *Elements of the Model*

The simplified nuclear model we use is briefly described here (NEA, 1994). Natural uranium is enriched for use in a LWR plant or used directly in a FBR plant. Production of nuclear power from LWR technology is assumed to be a linear function of the enriched uranium input. The enrichment process creates large quantities of depleted uranium, which cannot be used in the LWR but has economic value as an input in the FBR fuel mix. A key difference between the two technologies is the existence of joint

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<sup>21</sup> These reserves are recoverable at uranium prices of up to \$80/kg. Current prices are about \$30/kg. At substantially higher prices, seawater could be tapped for large amounts of the metal.

<sup>22</sup> To facilitate comparison, weapons programs require uranium enrichment of over 90%.

<sup>23</sup> This low price is partly due to the availability of weapons grade uranium and plutonium from military stockpiles of the US and the former Soviet Union. This higher grade uranium is blended down to provide reactor fuel. It currently provides almost 15% of the world's annual uranium supply.

<sup>24</sup> Our estimates, computed independently, are similar to those developed by an interdisciplinary MIT (2003) study (16 million tonnes).

products: several by-products from LWR production, the most important of which is plutonium, are used as inputs into FBR production. The LWR technology produces three different by-products: fissile waste which must be treated and stored, and plutonium and reprocessed uranium, both of which can be used in FBR reactors. These complementarities in material flows are shown in Fig. A1.

Consider a single deposit of low grade uranium ore starting at point A in the figure. This natural uranium could be enriched for use in a LWR plant or used directly without enrichment in a FBR facility. Define  $u_E^L$  as the instantaneous flow of natural uranium that is enriched and used in a LWR plant. Enriching the ore leads to the separation of uranium into enriched uranium ( $u_E^L$ ) and depleted uranium ( $u_D^L$ ). Let these ratios be  $\varepsilon$  and  $1 - \varepsilon$ , respectively, with  $0 < \varepsilon < 1$ . Then  $u_E^L = \varepsilon u_N^L$  and  $u_D^L = (1 - \varepsilon) u_N^L$ . Let  $q^L$  be the instantaneous production of energy (electricity) from LWR technology. We assume that it is a linear function of enriched uranium  $u_E^L = \alpha^L q^L$ . The LWR technology produces three different by-products - fissile waste which cannot be re-used and must be stored; plutonium, and reprocessed uranium. The last two can be re-used in the FBR. The amount of plutonium produced by LWR technology is denoted by  $Pu^L$  and is assumed to be proportional to the instantaneous production rate  $q^L$ , i.e.,  $Pu^L = \beta^L q^L$ . The amount of reprocessed uranium is similarly given by  $u_R^L = \xi^L q^L$ . The volume of wastes  $w^L$  generated by LWR technology is  $w^L = \gamma^L q^L$ , where  $\alpha^L, \beta^L, \gamma^L$  and  $\xi^L$  are given positive coefficients.

Let  $q^F$  be the corresponding production of energy from FBR technology. Again, we assume this to be a linear function of natural, depleted or reprocessed uranium, denoted respectively by  $u_N^F$ ,  $u_D^F$  and  $u_{Ri}^F$ , where the subscript  $i$  denotes input. For simplicity we assume that they are perfect substitutes so that  $u_N^F + u_D^F + u_{Ri}^F = \alpha^F q^F$ . This is shown by point B, where the stocks of natural, depleted and reprocessed uranium are merged into one. The unique feature of FBR technology is that it can reuse part of the plutonium produced. Therefore the choice of the breeding ratio, i.e., the input-output ratio of plutonium, denoted by  $\mu^F$  is endogenous. Thus the input of plutonium is given by  $Pu_i^F = \beta^F q^F$  and the output (denoted by subscript o) by  $Pu_o^F = \mu^F \beta^F q^F$ . The uranium and plutonium inputs in FBR must be used in fixed proportion  $k$ . Their complementarity is described by the relationship  $\frac{u_N^F + u_D^F + u_{Ri}^F}{Pu_i^F} \equiv k = \frac{\alpha^F}{\beta^F}$ .

The output of reprocessed uranium from FBR technology is denoted by  $u_{Ro}^F$ .<sup>25</sup> Its proportion is given by  $u_{Ro}^F = \xi^F q^F$ . Let  $w^F$  represent the amount of waste generated by the FBR technology. Then  $w^F = \gamma^F q^F$ . Again,  $\alpha^F, \beta^F, \gamma^F$  are positive constants. In summary, FBR technology uses uranium (natural, depleted and reprocessed) and plutonium as inputs, and produces energy, reprocessed uranium, plutonium and waste fissile material.

In summary, natural uranium is enriched before use in a LWR plant. This process increases the proportion of fissile uranium which sustains the chain reaction in a LWR reactor. The process of enrichment also generates large quantities of depleted (lower grade) uranium, which needs to be stockpiled, and has little economic value. However, this depleted uranium can be used in FBR technology, along with plutonium. Thus, the waste material from enrichment can be put to use in FBR reactors, producing yet more plutonium which can be used again.

### *Stock Dynamics*

We consider five distinct stocks of resources: natural uranium (in the ground), depleted uranium, reprocessed uranium, stockpiled plutonium, and nuclear wastes. The stock of uranium ore in the ground,  $U_N(t)$  is enriched and declines by the quantity extracted for LWR,  $u_N^L(t)$  and FBR,  $u_N^F(t)$  given by  $\dot{U}_N(t) = u_N^L(t) - u_N^F(t)$ . The stock of depleted uranium  $U_D(t)$  is augmented by the depleted uranium which is rejected from the enrichment process  $u_D^L(t) = (1 - \varepsilon)u_N^L(t)$ , and reduced by the extracted quantity to be used in FBR,  $u_D^F(t)$  given by  $\dot{U}_D(t) = (1 - \varepsilon)u_N^L(t) - u_D^F(t)$ . The stock of reprocessed uranium  $U_R(t)$  is augmented by the reprocessed uranium  $u_R^L(t)$  from LWR and  $u_{Ro}^F(t)$  from FBR, and reduced by the quantity  $u_{Ri}^F(t)$  to be used in FBR, and is given as  $\dot{U}_R(t) = u_R^L(t) - u_{Ri}^F(t) + u_{Ro}^F(t)$ . The stock of plutonium  $Pu(t)$  is augmented by the quantity  $\beta^L q^L(t)$  out of the LWR plant, minus the FBR input  $\beta^F q^F$ , and augmented by the plutonium created by FBR technology,  $\mu^F \beta^F q^F$  with  $\mu^F > 1$ . Now define  $\Delta$  as the time lag between the date at which the plutonium flow is extracted from the reactor and the date at which it is reintegrated into the plutonium stock for re-use, caused by the need to reduce the temperature of the mineral and other processing tasks. This is given by

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<sup>25</sup> The uranium input and output also need to be used in fixed proportions, satisfying the condition:

$$u_N^F(t) + u_D^F(t) + u_{Ri}^F(t) \equiv u_{Ro}^F(t) \left( \frac{\alpha^F}{\xi^F} \right).$$

$$\dot{Pu}(t) = Pu_o^F(t) - Pu_i^F(t) + Pu_o^L(t) = \beta^L q^L(t) - \beta^F q^F(t) + \beta^F \mu(t - \Delta) q^F(t - \Delta).$$

Finally, the flow of wastes from the two technologies,  $w^L$  and  $w^F$  are aggregated as follows:

$\dot{W}(t) = w^L(t) + w^F(t)$ . We assume zero radioactive decay of the nuclear waste because of the relatively short time horizon of the model.

### ***Nuclear Cost Functions***

Let  $m$  denote the average extraction cost of natural uranium, so that total extraction cost is  $m(u_N^L + u_N^F)$ .

Let  $m^S$  be the unit enrichment (separation) cost of uranium used in LWR. Then total enrichment cost equals  $m^S u_N^L$ . This enriched uranium is packaged and assembled before use as an input in LWR

production, at an average cost of  $m^L$ . Therefore, the total preparation cost of LWR uranium

is  $m^L u_E^L = m^L \epsilon u_N^L$ . The average cost of fuel reprocessing for LWR technology is denoted by  $m_R^L$ , so that the total cost is  $m_R^L [\beta^L + \gamma^L + \xi^L] q^L$ . Finally, the LWR reactor incurs an *in situ* operating cost of  $v^L q^L$ .

Let  $m_f^F$  and  $m_R^F$  denote the average preparation and reprocessing cost of FBR fuel, respectively. Then the total FBR fuel fabrication cost is  $m_f^F [u_N^F + u_D^F + u_{Ri}^F + r_i^F]$  and the total fuel reprocessing cost is  $m_R^F [\mu \beta^F + \gamma^F + \xi^F] q^F$ . The operating cost of FBR technology is given by  $v^F q^F$ . Each unit of depleted uranium is stockpiled at an average annual cost of storage  $s_{U_D}$ , so that the total storage cost is  $s_{U_D} U_D$ .

Similarly, let the respective annual unit cost of storage for reprocessed uranium and plutonium be  $s_{U_R}$  and  $s_{Pu}$  so that the corresponding storage costs are  $s_{U_R} U_R$  and  $s_{Pu} Pu$ . Finally the annual unit cost of storage for reprocessed uranium is  $s_w$  so that the total cost is given by  $s_w [\gamma^L q^L + \gamma^F q^F]$

### *Optimization of the Nuclear Model*

Production of nuclear energy is optimized by choosing the instantaneous amount of power generated by the two technologies,  $q^L(t)$  and  $q^F(t)$  and the breeding ratio  $\mu^F(t)$ , to maximize a social surplus function, net of total costs. Denote the instantaneous gross surplus as  $S(t) = S(q^L(t) + q^F(t))$ . With a constant social rate of discount  $\delta$ , we have

$$\begin{aligned}
\text{Max}_{\{q^L(t), \{q^F(t)\}, \{\mu^F(t)\}} \int_0^\infty \{ & S(q^L(t) + q^F(t)) - m[u_N^L(t) - u_{Nt}^F(t)] - [m^S + \varepsilon m_f^L] u_{Ni}^L(t) \\
& - m_R^L [\beta^L + \gamma^L + \xi^L] q^L(t) - v^L q^L(t) \\
& - m_f^F [u_N^F(t) + u_D^F(t) + u_{Ri}^F(t) + r_i^F(t)] \\
& - m_R^F [\mu(t) \beta^F + \gamma^F + \xi^F] q^F(t) - v^F q^F(t) \\
& - s_{U_D} U_D(t) - s_{U_R} U_R(t) - s_{Pu} \cdot Pu(t) \\
& - s_W [\gamma^L q^L(t) + \gamma^F q^F(t)] e^{-\delta t}
\end{aligned}$$

subject to

$$\begin{aligned}
\dot{U}_N(t) &= -u_N^L(t) - u_N^F(t), & U_{N_0} > 0 \text{ given, } U_N(t) &\geq 0 \\
\dot{U}_D(t) &= -u_D^L(t) - u_D^F(t), & U_{D_0} > 0 \text{ given, } U_D(t) &\geq 0 \\
\dot{U}_R(t) &= u_R^L(t) - u_{Ri}^F(t) + u_{Ro}^F(t), & U_{R_0} > 0 \text{ given, } U_R(t) &\geq 0 \\
\dot{P}u(t) &= r_{oi}^L(t) - r_i^F(t) + \mu^F(t - \Delta) r_i^F(t - \Delta), & P_{u_0} > 0 \text{ given, } Pu(t) &\geq 0 \\
\dot{W}(t) &= \gamma^L q^L(t) + \gamma^F q^F(t), & W_0 > 0 \text{ given, } W(t) &\geq 0
\end{aligned}$$

$$\bar{\mu}^F - \mu^F(t) \geq 0 \text{ and } \mu^F(t) - \underline{\mu}^F \geq 0,$$

$$q^C(t) \geq 0,$$

$$q^F(t) \geq 0,$$

$$u_N^L(t) = \varepsilon^{-1} \alpha^L q^L(t) \geq 0,$$

$$u_D^L(t) = (1 - \varepsilon) \varepsilon^{-1} \alpha^L q^L(t) \geq 0,$$

$$r_{oi}^L(t) = \beta^L q^L(t) \geq 0,$$

$$r_i^F(t) = \beta^F q^F(t) \geq 0,$$

$$u_{Ri}^L(t) = \xi^L q^L(t) \geq 0,$$

$$u_{Ro}^F(t) = \xi^F q^F(t) \geq 0,$$

$$u_N^F(t) + u_D^F(t) + u_{Ri}^F(t) \equiv r_i^F(t) \cdot \frac{\alpha^F}{\beta^F} = \alpha^F q^F(t),$$

$$u_N^F(t) + u_D^F(t) + u_{Ri}^F(t) \equiv u_{Ro}^F(t) \left( \frac{\alpha^F}{\xi^F} \right).$$

The necessary conditions are available separately from the authors.

## Appendix B. Data

### *Nuclear Data*

Aggregate estimated reserves of uranium ore, including those already discovered and those recoverable at a higher cost, have been estimated to be of the order of 15.4 million tons (OECD, 2000a). We consider a single grade of uranium.<sup>26</sup> The actual cumulative production of nuclear power since the technology was deployed now exceeds 34,000 TWh (1TWh= $10^9$  KWh). This implies that approximately one million tons of plutonium and 0.1 million tons of fissile waste have been produced, including discharged uranium and other fission by-products.<sup>27</sup> These values are used as initial stocks. Since reprocessed uranium is only used for mixed oxide fuels not considered in the paper, its initial stock is assumed to be zero.

LWR technology is modeled on the European Pressurized Reactor (EPR) with a capacity of 1450 MW, producing 11.46 TWh of power annually. The spent fuel discharge consists of 19.132 tons of uranium, 0.271 tons of plutonium, 0.0417 tons of minor actinides and 1.369 other tons of fission products (see Charpin, 2000). After reprocessing and cooling, each TWh of electrical energy generates 23 kg of plutonium and 120.5 kg of wastes.<sup>28</sup>

FBR technology is based on the European Fast Reactor (EPR) with a capacity of 1000 MW, producing 8.76 TWh of power. This plant requires 11.7 tons of uranium and 1.5 tons of plutonium annually. The spent fuel discharge consists of 10.4 tons of uranium, one ton of fission products and 0.3 tons of plutonium, which is recycled back into the plant.<sup>29</sup>

Long-run cost estimates for nuclear power are obtained from NEA (1994, 2001, 2002). We have simplified the specification of the technology and regrouped some stages whose costs are low or which involve a simple transformation of products without any storage. The cost of reprocessing or storing joint products such as reprocessed uranium from LWR plants which can be used in FBR technology are suitably apportioned between the two technologies. For simplicity, we assume constant returns to scale technologies and unit costs that are fixed over time. It is likely that technological change and the costs of labor, capital and materials may alter relative costs over time. It is difficult to predict these changes *ex*

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<sup>26</sup> The additional benefits of modeling with multiple uranium ores is low, since the total stock is small enough to be exhausted by use in LWR plants within a few decades.

<sup>27</sup> During this period,  $(1 - \mathcal{E})$  or 0.917 million tons of depleted uranium have been stockpiled (OECD, 2000a).

<sup>28</sup> LWR waste production decreases with FBR operation because of reprocessing of spent fuels.

<sup>29</sup> Further details on the energy content of fissile material is available in tabular form from the authors and from Hore-Lacy (2003) and Brown (2002).

*ante*, but we partly address this issue by applying across the board technology-induced cost reductions.

The unit cost of extraction of uranium oxide and its conversion to uranium hexafluoride is assumed to be \$60/kg of uranium. The separation and enrichment stage involves processes that add significant value to the mineral.<sup>30</sup> The cost of enrichment is taken as \$76/kg of uranium. The fuel fabrication stage also represents a significant part of the fuel cycle cost and depends largely on the type of reactor. It is assumed to be \$305/kg for LWR fuels, and a high \$3,050/kg for FBR fuels, partly because of additional safety measures associated with the handling of large amounts of plutonium. The unit cost of reprocessing spent fuel is assumed to be \$500/kg for LWR and \$1,600/kg for FBR.

Investment and direct operating costs represent the largest fraction of total costs in electricity generation. They are assumed to be \$5.614/mmBtu of delivered electricity for LWR, and 50% higher for FBR. The disposal cost of depleted uranium is taken as \$2.5/kg. The cost of interim storage of plutonium is a high \$1,000/kg, because of its toxicity. The cost of conditioning of the waste and long-term geological storage is assumed to be independent of the source of the waste material (LWR or FBR) and is taken to be \$500/kg. Table B1 provides a summary of the cost estimates.

### ***Other Energy Data***

The baseline sectoral elasticities and technical coefficients are taken from Nordhaus (1979) and Chakravorty *et al.* (1997) respectively, and they are given in Table B2. The extraction costs by resource and grade are given in Table B3.<sup>31</sup> These reserves include known unconventional reserves (e.g., oil and gas in shales and tar sands).<sup>32,33</sup> The cost of solar photovoltaic energy is taken to be 25 cents/kWh.<sup>34</sup> The carbon ceiling is in the form of the concentration of atmospheric CO<sub>2</sub> equal to 550 parts per million.

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<sup>30</sup> Separation produces a large quantity of stockpiled depleted uranium. Recall that this stock is waste in a LWR operation, but is an important source of uranium for FBR technology.

<sup>31</sup> Several deposits of a resource each with constant extraction cost may approximate a cost function that increases with cumulative extraction, as assumed in several studies (e.g., Nordhaus and Boyer, 2000).

<sup>32</sup> For instance, recoverable shale oil reserves are estimated to be of the same order of magnitude as oil reserves in the two most oil-rich OPEC countries - Saudi Arabia and Iraq, each with approximately 230-260 billion barrels (The New York Times, 2003). Hydrocarbon reserves in the ocean bed are estimated to be several orders of magnitude higher, albeit available at a significantly higher cost.

<sup>33</sup> The unit cost of oil per barrel is \$10 for the higher grade (IEA, 2000) (\$1/bbl= 0.18\$/mmBtu) and \$21 (Chakravorty *et al.*, 1997) for the lower grade. The unit costs for the higher grade of gas is \$1.75/mmBtu, and for the lower grade, \$2.76/mmBtu, which corresponds to the third quartile of the distribution of gas extraction costs (Greene, 1999). The unit cost for the three grades of coal are \$1.36, 2.37 and 5.08 per ton respectively (\$1/mmBtu=\$15.02/ton of coal).

<sup>34</sup> A vector of conversion costs from each resource to each demand is added to the extraction cost. The total cost is shown in Table B3. Sectoral conversion efficiencies are taken from (Chakravorty *et al.*, 1997, Table 5).

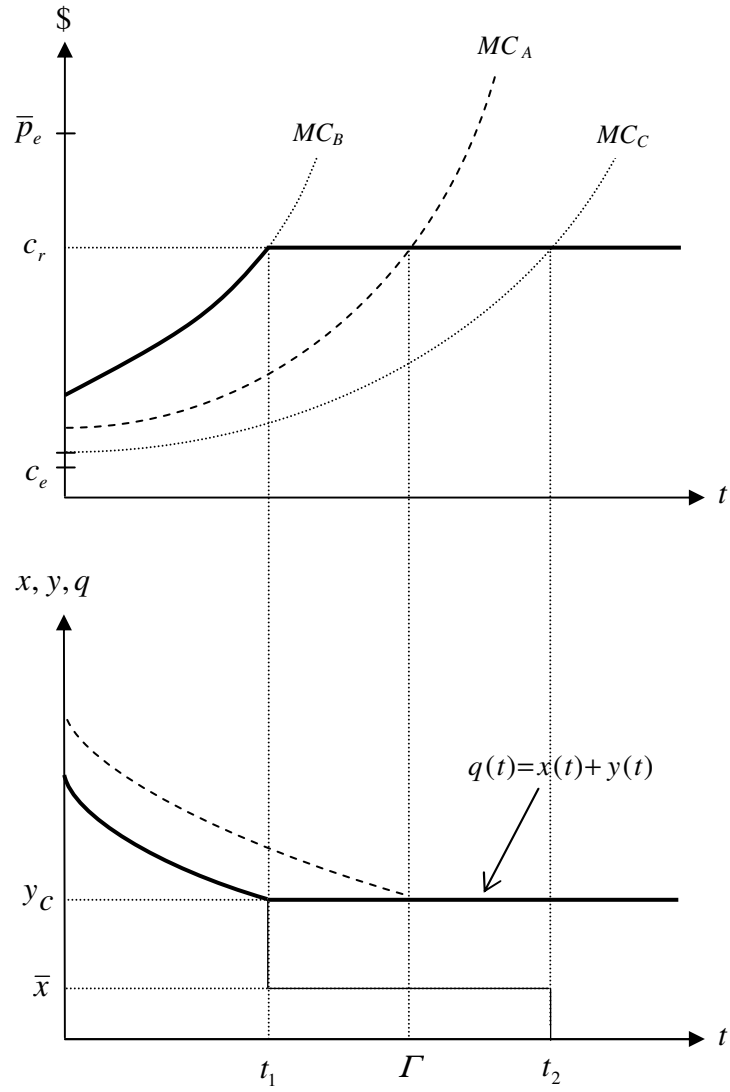
Various climate studies have used 450-650 ppm (e.g., Manne and Richels, 2002).<sup>35</sup> In the paper we perform a sensitivity analysis over this range. Atmospheric concentrations are computed using carbon emission rates from Nordhaus and Boyer (2000), after adjusting for the different time intervals in our model.<sup>36</sup> The discount rate is assumed to be 2%.

The GDP growth rate assumptions are the same for all the models except model E. In the former, we assume a steady decline in GDP growth over time, as shown in Fig. B1. As in Chakravorty *et al.* (1997), global GDP is expected to grow over time at an initial rate of 3% every year. This growth rate declines at the rate of 10% every decade. For model E, we consider an increase in the GDP growth rate in the near term followed by a decline, which reflects more accurately the expected sharp decline in population and the inevitable satiation in per capita energy use in the long run (see Fig. B1). The rate of GDP growth declines geometrically at 5% per decade until 2060, and at 10% per decade beyond this period. Decadal GDP growth is close to zero (less than 0.5%) around 2110 and 0.1% around year 2250. Fig. B2 shows the underlying population and GDP per capita assumptions. World GDP has been calibrated so that per capita GDP asymptotically reaches the U.S. level of \$25,500.

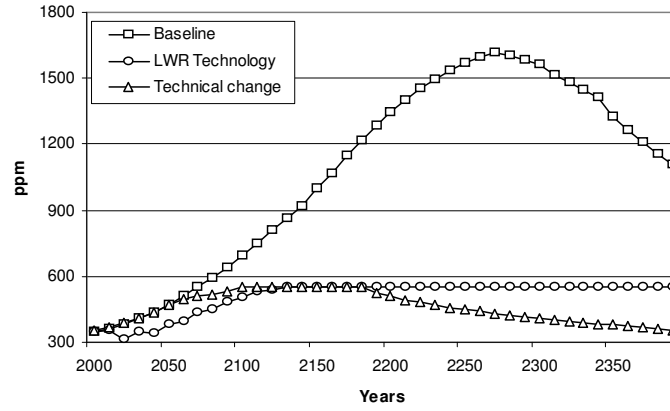
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<sup>35</sup> Current CO<sub>2</sub> concentration levels are approximately 370 ppm. A target of 550 ppm is expected to produce some warming but without catastrophic effects (Hoffert *et al.*, 2002).

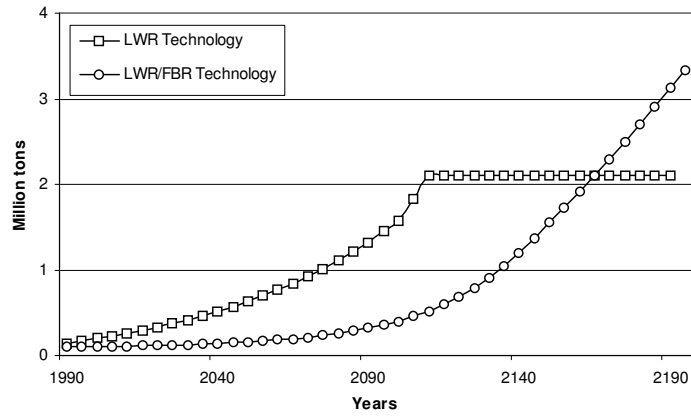
<sup>36</sup> In the case of modern nuclear technology, the algorithm is run on 5 year intervals, since reprocessing of the spent fuel takes approximately 5 years. Since Nordhaus and Boyer use 10 year intervals, we adjusted their emission rates to correspond to our 5 year intervals.



**Fig. 1: Both the Polluting Fossil Fuel and the Clean Renewable are used at the Ceiling**



**Fig. 2: Time Path of Carbon Concentration**



**Fig. 3: Cumulative Stock of Nuclear Wastes**

**Table 1: Resource Use in the Baseline Model without Nuclear Power**

Baseline, Fossil Fuels, no Nuclear Power				
<i>Decade</i>	<i>Elec.</i>	<i>Transp.</i>	<i>Resid. / Com.</i>	<i>Ind.</i>
1990-99	Coal I	Oil I	Gas I	Gas I
2000-09	Coal I	Oil I	Gas I	Gas I
...	...	...	...	...
2040-49	Coal I	Oil I	Gas II	Gas I-II
2050-59	Coal I	Oil I	Gas II	Gas II
2060-69	Coal I	Oil I-II	Gas II	Gas II
2070-79	Coal I	Oil II	Gas II	Gas II
2080-89	Coal I	Oil II	Gas II	Gas II / Coal I
2090-99	Coal I-II	Oil II	Gas II	Coal II
2100-09	Coal II	Oil II	Gas II	Coal II
...	...	...	...	...
2140-49	Coal II	Oil II / Coal II	Coal II	Coal II
2150-59	Coal II	Coal II	Coal II	Coal II
...	...	...	...	...
2210-19	Coal III	Coal II-III	Coal II	Coal III
2220-29	Coal III	Coal III	Coal III	Coal III
...	...	...	...	...
2260-69	Coal III / Solar	Coal III	Coal III	Coal III
2270-79	Solar	Coal III	Coal III	Coal III
2280-89	Solar	Coal III	Coal III	Coal III
2290-99	Solar	Solar	Coal III	Coal III
...	...	...	...	...
2330-39	Solar	Solar	Solar	Solar
...	...	...	...	...

Note: Higher numerals denote costlier grades

**Table 2: Percentage Shares of Primary Energy Supply**

<i>Scenarios</i>	<i>Oil</i>			<i>Gas</i>			<i>Coal</i>			<i>Nuclear</i> <sup>1</sup>		
	2020	2050	2100	2020	2050	2100	2020	2050	2100	2020	2050	2100
Baseline	16	13	12	71	70	38	16	18	50	-	-	-
LWR	14	14	44	70	70	41	11	11	4	5	6	11
LWR/FBR	13	13	11	71	71	53	8	4	0	7	13	44
Tech Prog	23	33	6	71	37	25	0	16	0	7(0)	10(5)	5(54)
Stagnation in Demand	13	13	11	72	72	51	8	4	0	7	11	38

<sup>(1)</sup> Solar shares are in brackets.

**Table 3: Resource Use under a Pollution Target with (a) LWR and (b) LWR, FBR**

Baseline, LWR, 550 ppm target					Baseline, LWR-FBR, 550 ppm target				
Decade	<i>Elec.</i>	<i>Transp.</i>	<i>Resid. / Com.</i>	<i>Ind.</i>	Decade	<i>Elec.</i>	<i>Transp.</i>	<i>Resid. / Com.</i>	<i>Ind.</i>
1990-99	Oil I / LWR	Oil I	Gas I	Gas I	1990-99	Gas I / LWR	Oil I	Gas I	Gas I
2000-09	Coal I / LWR	Oil I	Gas I	Gas I	2000-09	Gas-Coal I / LWR	Oil I	Gas I	Gas I
...	...	...	...	...	...	...	...	...	...
2040-49	Coal I / LWR	Oil I	Gas I	Gas I-II	2040-49	Coal I / LWR	Oil I	Gas I-II	Gas I-II
2050-59	Coal I / LWR	Oil I	Gas II	Gas II	2050-59	Coal I / LWR	Oil I	Gas II	Gas II
2060-69	Coal I / LWR	Oil I-II	Gas II	Gas II	2060-69	LWR	Oil I-II	Gas II	Gas II
2070-79	Coal I / LWR	Oil II	Gas II	Gas II	2070-79	LWR / FBR	Oil II	Gas II	Gas II
2080-89	Coal I / LWR	Oil II	Gas II	Gas II / Oil II	2080-89	LWR / FBR	Oil II	Gas II	Gas II
2090-99	Coal I / LWR	Oil II	Gas II	Oil II	2090-99	FBR	Oil II	Gas II	Gas II / LWR / FBR
2100-09	Coal I / LWR	Oil II	Gas II	Oil II	2100-09	FBR	Oil II / Gas II	Gas II	Gas II / LWR / FBR
2110-19	Solar / LWR	Oil II	Gas II	Coal I	2110-19	FBR	Oil II / Gas II	Gas II / FBR	LWR / FBR
2120-29	Solar / LWR	Oil II	Gas II	Coal I / Gas II	2120-29	FBR	Oil II	Gas II / FBR	FBR
2130-39	Solar	Oil II	Gas II	Coal I / Gas II	2130-39	FBR	Oil II	Gas II / FBR	FBR
2140-49	Solar	Oil II / Sol.	Gas II	Coal I	2140-49	FBR	Oil II	FBR	FBR
2150-59	Solar	Oil II / Sol.	Gas II	Coal I	2150-59	FBR	Oil II	FBR	FBR
2160-69	Solar	Solar	Gas II	Coal I	2160-69	FBR	Oil II / Coal I	FBR	FBR
2170-79	Solar	Solar	Gas II	Coal I	2170-79	FBR	Coal I / FBR	FBR	FBR
2180-89	Solar	Solar	Coal I / Sol.	Coal I	2180-89	FBR	Coal I / FBR	FBR	FBR
2190-99	Solar	Solar	Coal I-II / Sol.	Coal I	2190-99	FBR	Coal I / FBR	FBR	FBR
2200-09	Solar	Solar	Coal II / Sol.	Coal II	2200-09	FBR	Coal I / FBR	FBR	FBR
...	...	...	...	...	...	...	...	...	...

(a)

(b)

Note: Higher numerals denote costlier grades

**Table 4: Resource Use with a Carbon Target and All-round Technical Progress**

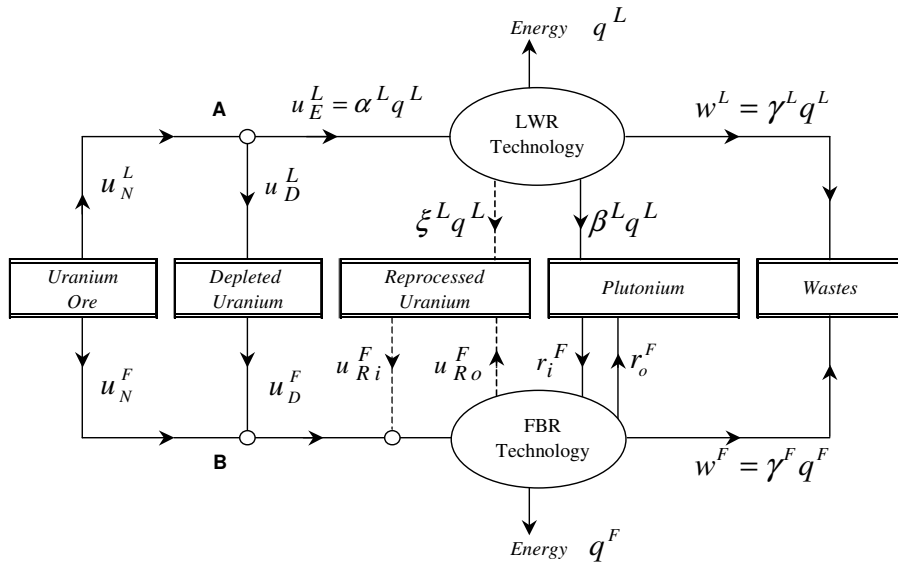
Exogenous Decline in Costs, 550 ppm Target				
Decade	<i>Elec.</i>	<i>Transp.</i>	<i>Resid. / Com.</i>	<i>Ind.</i>
1990-99	Oil I / LWR	Oil I	Gas I	Gas I
2000-09	Oil I / LWR	Oil I	Gas I	Gas I
2010-19	Oil I / LWR	Oil I	Gas I	Gas I
2020-29	Oil-Coal. I / LWR	Oil I	Gas I	Gas I
2030-39	Coal I / LWR	Oil I / LWR	Gas I	Gas I
2040-49	Coal I	Oil I / LWR / Sol.	Gas I	Gas I / Oil I
2050-59	Coal I	Sol. / LWR-FBR	Gas II	Oil I / Gas II
2060-69	Solar	Sol. / LWR-FBR	Gas II	Gas II
2070-79	Solar	Sol. / LWR-FBR	Gas II	Gas II / Oil II
2080-89	Solar	Sol. / LWR-FBR	Gas II	Oil II
2090-99	Solar	Sol. / LWR-FBR	Gas II	Oil II
2100-09	Solar	FBR / Solar	Gas II	Oil II / Sol.
2110-19	Solar	Solar	Gas II	Solar
2120-29	Solar	Solar	Gas II / Sol.	Solar
2130-39	Solar	Solar	Solar	Oil II / Sol.
...	...	...	...	...
2180-89	Solar	Solar	Solar	Solar
...	...	...	...	...

Note: Higher numerals denote costlier grades

**Table 5: Scarcity Rent of Uranium (\$/kgU) and Shadow Price of Carbon in Year 2000 (\$/ton)**

Scenarios	Carbon Targets (ppm)	Scarcity Rent of Uranium (\$/kg)	Shadow Price of Carbon (\$/ton)
LWR Technology	450	1542	207
	550	1542	40
	650	1143	21
LWR Technology and Doubling of Uranium Stock <sup>1</sup>	550	697	32
	450	176	137
Both Nuclear Technologies	550	160	26
	650	163	3
	450	17	10
Both Nuclear Technologies and Technical Progress	550	95	22

<sup>(1)</sup> 30.8 million tons.



**Fig. A1: Flow of Materials in the Nuclear Technology**

**Table B1. Unit Costs for the Nuclear Technology <sup>1</sup>**

<i>Cost parameters</i>	<i>LWR</i>	<i>FBR</i>
Extraction $m$	60	60
Enrichment $m^S$	76	-
Fuel Fabrication $m_f^L, m_f^F$ ,	305	2,600
Operating $v^L, v^F$	5.614	8.421
Processing $m_R^L, m_R^F$	500	2,000
Depleted Uranium Storage $s_{U_D}$	3.5	-
Reprocessed Uranium Storage $s_{U_R}$	3.6	3.6
Plutonium Storage $s_{Pu}$	1,000	1,000
Waste Disposal $s_W$	400	400

<sup>(1)</sup> in \$/kg

**Table B2. Sectoral Demand Parameters**

<i>Sector <math>i</math></i>	<i>Constant parameter <math>A_j</math></i>	<i>Price elasticity <math>\alpha_j</math></i>	<i>Income elasticity <math>\beta_j</math></i>
1. Electricity	0.015927	-0.65	0.92
2. Transportation	1.699235	-1.28	0.81
3. Residential / Commercial	0.006730	-0.79	1.08
4. Industry	0.091866	-0.52	0.76

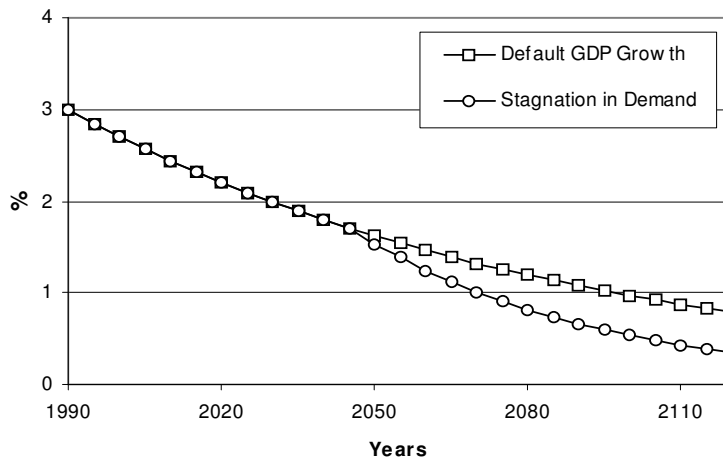
**Table B3. Resource Stocks <sup>1</sup> and Extraction Plus Conversion Costs by Sector (\$/mmBtu)**

	<i>Elec</i>	<i>Transp</i>	<i>Resid</i>	<i>Ind</i>	<i>Extraction Costs <sup>2</sup></i>	<i>Stocks</i>
<i>Oil Grade I</i>	13.21	68.65	15.81	5.28	0.60	2,100.91
<i>Oil Grade II</i>	18.34	74.05	17.84	7.60	3.47	3,851.67
<i>Natural gas Grade I</i>	12.88	78.15	9.48	4.63	0.92	14,218.50
<i>Natural gas Grade II</i>	16.31	81.49	10.73	6.06	2.76	35,072.30
<i>Coal Grade I</i>	15.59	113.34	21.71	11.10	1.51	989.79
<i>Coal Grade II</i>	18.85	117.22	23.10	12.49	2.37	5,802.25
<i>Coal Grade III</i>	27.93	128.06	26.97	16.36	5.08	4,369.58
<i>Solar</i>	73.20	213.07	96.55	87.71		-
<i>Electricity <sup>3</sup></i>	-	126.95	10.43	1.59		-

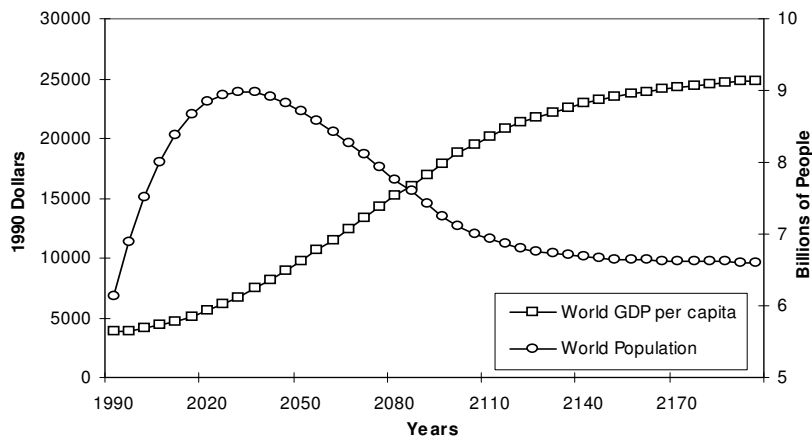
<sup>(1)</sup> The stock of oil, coal and gas are in billion barrels, tons and billion mmBtu, respectively.

<sup>(2)</sup> Extraction costs are in \$/mmBtu.

<sup>(3)</sup> These are conversion costs of electricity to each sector. For example, the total cost of nuclear in residential is the cost of nuclear electricity plus the conversion cost of electricity to residential energy.



**Fig. B1: GDP Growth Rate Assumptions**



**Fig. B2: Stagnation in Demand Growth: GDP per capita and Population Assumptions**