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## Specialization and non-renewable resources: Ricardo meets Ricardo

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### Abstract

The one-demand Hotelling model fails to explain the observed specialization of non-renewable resources. We develop a model with multiple demands and resources to show that specialization of resources according to demand is driven by Ricardian comparative advantage while the order of resource use over time is determined by Ricardian absolute advantage. An abundant resource with absolute advantage in all demands must be initially employed in all demands. When each resource has an absolute advantage in some demand, no resource may be used exclusively. The two-by-two model is characterized. Resource and demand-specific taxes are shown to have significant substitution effects.

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## 1. Introduction

The modern theory of resource economics (Hotelling, 1931; Herfindahl, 1967), which traces its roots to Ricardo's theory of the mine,<sup>1</sup> has largely focused on the problem of one resource and a single demand. However, casual empirical observation suggests that different non-renewable resources (e.g., oil, coal) and grades (e.g., crude oils from Saudi Arabia and Alaska) are being extracted simultaneously to satisfy distinct energy demands. For example, Table 1 shows the composition of global energy supply by sector and resource for a recent year (2000). Notice that crude oil and petroleum products are the fuels of choice in the transportation sector, coal in industry and natural gas in industrial and residential uses. Coal, nuclear, natural gas and hydroelectric energy supply the bulk of electricity, an intermediate good used mainly in the industrial and residential sectors. The single-demand model does not adequately explain this simultaneity and specialization of resources in specific sectors.

Not only are multiple non-renewable resources being extracted simultaneously, but the pattern of resource substitution is induced by a host of economic factors (e.g., demand growth, the discovery of new reserves, and technological change) that may vary between sectors and between resources. A recent International Energy Agency (IEA) study of energy demand until 2020 suggests that in the OECD countries, growth in consumption in the transport sector is entirely accounted for by oil, while in the residential, industry and electricity sectors, oil continues to lose market share to other fuels, especially natural gas (IEA, 2000). In particular, natural gas, in recent years, has become competitive in the electricity sector because of the advent of combined cycle gas turbine technology. In the past, electricity was generated predominantly from coal, but new power plants today mostly use natural gas, except in countries with large indigenous reserves of coal. This trend is significant enough that the IEA expects global energy supplies from natural gas to surpass that of coal around 2010. The shift from coal to natural gas is also evident in the industry and residential sectors. The transition from coal and oil to the cleaner natural gas and the switch from oil to coal in industry after the OPEC oil price shocks of the seventies suggests that correctly specified empirical models that study the evolution of long-run resource price movements need to allow for fuel switching within and between sectors.<sup>2</sup>

The class of models following from Hotelling (1931) and Herfindahl (1967), who extended Hotelling to multiple grades of a resource (still with a single demand), cannot explain this apparent specialization of resources in particular demands because in the single demand framework, different resources are indistinguishable from different grades of the same resource. Due to this restriction, empirical tests of

<sup>1</sup>Chapter 3 in Ricardo (1912), first edition, 1817.

<sup>2</sup>Another reason why a multiple (rather than a single) demand model may be a more appropriate framework for analysis is the significant difference in growth characteristics among sectors. For example, annual average growth in global demand in the electricity (2.8%) and transport sectors (2.4%) is markedly higher than that in other uses such as residential and industry (1.8%), according to IEA (2000).

Table 1  
World total final energy consumption by sector (in MTOE): 2000

Sectors	Coal	Crude oil & products	Natural gas	Nuclear, hydro & others <sup>a</sup>	Total	Percent share
Industry	411.5	592.4	491.1	691.1	2186.1	31.6
Transportation	5.9	1701.4	53.7	27.8	1788.8	25.9
Residential & others	128.9	655.9	570.3	1575.3	2930.4	42.5
TFC	546.3	2949.7	1115.1	2294.2	6905.3	100
Percentage share	7.9	42.7	16.2	33.2	100	

*Glossary:* MTOE, Million tons of oil equivalent; TFC, total final consumption.

*Note:* Electricity consumption is not reported separately since it is an intermediate product. In 2000, 1322.6 MTOE of electricity was produced out of which 42.2% was used in industry and 56% in the ‘Residential and Others’ sector. Electricity was mainly produced from coal (39%), nuclear, natural gas and hydro (approximately 17% each) and oil (8%).

*Source:* Adapted from International Energy Agency (2002).

<sup>a</sup>Includes combustible renewables, industrial waste, solar, wind, etc.

resource prices based on the Hotelling theory may be mis-specified (see e.g., Halvorsen and Smith, 1991).<sup>3</sup> In this paper, we extend the above class of Hotelling–Herfindahl models to a general multiple demand framework that allows for specialization of resources and substitution across demands based on notions of absolute and comparative advantage. We show that this extension generates an equilibrium sequence of resource extraction and energy prices over time that is quite distinct from those derived from standard Hotelling theory.

There are several papers that have extended Hotelling to multiple resources and demands, beginning with Herfindahl (1967) as noted earlier, who extended the Hotelling model to  $m$  resources and a single demand. Kemp and Long (1980), Lewis (1982), and Amigues et al. (1998) have generalized the basic  $m \times 1$  model by developing sufficient conditions under which Herfindahl’s ‘least-cost-first’ principle (the ordering of resource extraction by their unit cost) holds and conditions under which it may not hold.

Nordhaus (1973, 1979) pioneered the extension to the  $m \times n$  ( $m$  resources and  $n$  demands) case in an applied study concerning the long-run tendency of energy prices. Chakravorty et al. (1997) generalized and applied the Nordhaus framework to examine the effect of exogenous changes in the price of the backstop technology on fossil fuel extraction and carbon emissions over time. These applied studies do not develop the analytics of the  $m \times n$  model. Chakravorty and Krulce (1994) provided a theoretical treatment of a  $2 \times 2$  case (two resources and two demands) when one resource has absolute advantage over the other. However, as will become

<sup>3</sup>In his Richard T. Ely address to the American Economic Association, Arnold Harberger (1993) expressed the same sentiment when he suggested that resource economics is a prominent example of economic theory failing to live up to the demands of its practitioners. The gap between theory and practice can be partially closed by developing a theory of multiple resources that are imperfect substitutes in meeting various resource demands.

clear in this paper, their characterization of the  $2 \times 2$  model is a special case.<sup>4</sup> The objective of the present paper is to develop the general theory for the  $m \times n$  model and then to completely characterize the  $2 \times 2$  case.

The  $m \times n$  framework affords an analytical distinction between resources and resource grades. We assume that there is a constant extraction cost for each grade but that different grades of the same resource may have different extraction costs. We follow Nordhaus in assuming that energy demands in different sectors are independent.<sup>5</sup> Solutions to an infinite horizon maximization problem yield equilibrium relationships for a given resource in a given demand in terms of the scarcity rent and cost characteristics of the resource.<sup>6</sup>

We show that patterns of resource use can be characterized by stages according to which resources are used in what demands. We exploit Ricardian notions of absolute and comparative advantage in characterizing dynamic patterns of resource specialization. For example, if one resource has a Ricardian absolute advantage over all other resources and is sufficiently abundant, it will be used exclusively for all end uses in the first stage. However, when each resource has an absolute advantage in some demand in a  $m \times m$  model, a resource can never supply all demands, however abundant it may be. A strictly inferior resource will be used in all end uses in the final stage, regardless of its initial abundance. The two-resource two-demand case is completely characterized under conditions in which one resource has an absolute advantage in both uses, and when both resources have absolute advantage in some demand.

Our results suggest that absolute advantage leads to dynamic specialization while comparative advantage results in intersectoral specialization. A resource that is abundant and has absolute advantage in all demands may be extracted for a demand even though it does not have comparative advantage in that use. However if each

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<sup>4</sup>They developed a  $2 \times 2$  model in an infinite horizon framework where one resource (oil) has absolute advantage over the other in both demands. They showed that under these specific conditions, it will always be the case that a more expensive resource will be used for a finite time interval even though the cheaper resource is not exhausted, violating the well known 'least-cost-first' principle proposed by Herfindahl. The objective of their paper was to show that this principle need not hold in a multiple demand, partial equilibrium framework. However, they did not proceed to develop the full implications of the multiple demand framework.

<sup>5</sup>This assumption of independence is supported by the fact that seasonal shocks such as the effect of summer driving on demand for transportation energy or a cold winter on residential energy demand tend to be sector-specific.

<sup>6</sup>Gaudet et al. (2001) have examined a somewhat analogous general problem in which solid wastes are transported from urban centers to spatially distributed landfills. In their model, landfill capacity is exhaustible, and landfills are differentiated by transportation costs from each city. The landfill model is similar to ours in that transportation costs from cities to landfills can be thought of as conversion costs of resources to demands. However, resources are differentiable by class (oil, coal) and by grade (different grades of oil) while landfills are homogenous except for their location. While Gaudet et al. focus on the role of set up costs, we characterize patterns of optimal resource use according to principles of absolute and comparative advantage of a resource. Thus their work although in a spatial urban economics setting, may be thought of as a special case of our model in which each resource is of a different class. In later work it may be useful to develop a general model with both resource-independent transportation costs and demand-specific conversion costs.

resource has absolute advantage in a given demand, then specialization is likely to occur on the basis of comparative advantage. Ricardian comparative advantage thus provides a basis for developing the dynamic Ricardian/Hotelling theory of resource rents.

We show that taxes on a resource or on a sector (e.g., transportation) in the multiple demand model may have effects that are quite different than in the standard Hotelling model, which only predicts lower resource use over time. For example, a tax on coal may lead to both sectors switching to oil, while a tax on the transportation sector may lead to both sectors using oil, so that aggregate oil consumption may increase. A resource and sector-specific tax (such as a gasoline tax) may lead to a complete switch in resource use between sectors. That is, if oil was being used for transportation and coal for electricity *ex ante*, coal would be used for transportation and oil for electricity after the tax. Finally, predictions on energy price paths can be obtained directly from the sum of extraction plus conversion costs of resources. In the case of ‘clean’ oil and ‘dirty’ coal, we show that sectoral energy price differentials must decline over time, in sharp contrast to standard Hotelling theory, which predicts prices in both sectors to increase at the rate of discount.

Section 2 develops the general  $m \times n$  Hotelling model. Section 3 characterizes the  $2 \times 2$  case. Section 4 concludes the paper.

## 2. The $m \times n$ model

Consider a finite set of resources  $R$  (e.g. various grades of oil, coal, natural gas, etc.) and a finite set of uses for these resources defined by the set  $U$  (such as electricity, heating, transportation, etc.). The available stock of resource  $i \in R$ , assumed known, is  $Q_i(t_0) > 0$  which can be extracted at a constant unit cost of  $c_i \geq 0$ . Derived demand for energy in use  $j \in U$  is a strictly positive, bounded, continuous, strictly decreasing function of price,  $D_j(p)$  with  $\int_0^\infty D_j(p) dp < \infty$ . This last restriction implies a finite consumer surplus and is useful in guaranteeing a solution to the problem. Energy for the same use generated from different resources is assumed to be identical and the differences between resources subsumed in conversion costs. For example, coal can either be liquefied and used to produce a gasoline substitute, or car engines can be designed to run on coal, whichever is cheaper. The conversion cost of coal for use in transportation is then derived from the lesser of these costs. Conversion costs are resource and demand-specific such that there is a vector mapping from each resource to the set of demands. They are denoted by  $v_{ij} \geq 0$ , which is the cost of converting a unit of resource  $i$  to use  $j$ . Energy losses, such as frictional, heat or handling losses in the conversion process, are assumed to be incorporated into the cost of conversion and already netted out of the resource endowments.<sup>7</sup> The demand relationship is in terms of ‘delivered’ energy units. We define the *net cost* of supplying resource  $i$  to demand  $j$  as  $w_{ij} \equiv c_i + v_{ij}$ .

<sup>7</sup>In empirical applications, efficiency losses can be made explicit (see e.g. Nordhaus, 1979; Chakravorty et al., 1997).

The social planner chooses the quantity of each resource supplied to each demand. We denote by  $q_{ij}(t)$  the quantity of resource  $i$  supplied to demand  $j$  at time  $t$ . The problem is to determine the resource allocation that maximizes the present value of net social benefit. Given a discount rate  $r > 0$ , this can be posed as the optimal control problem: choose  $q_{ij}(t)$  for  $i \in R$  and  $j \in U$  to maximize

$$\int_{t_0}^{\infty} e^{-rt} \left[ \sum_{j \in U} \int_0^Z D_j^{-1}(x) dx - \sum_{i \in R} \sum_{j \in U} w_{ij} q_{ij}(t) \right] dt \tag{1}$$

subject to

$$q_{ij}(t) \geq 0, \quad Q_i(t) \geq 0 \quad \text{for } i \in R \text{ and } j \in U, \tag{2}$$

and

$$\dot{Q}_i(t) = - \sum_{j \in U} q_{ij}(t) \quad \text{for } i \in R, \tag{3}$$

where  $Z = \sum_{i \in R} q_{ij}(t)$ , aggregate energy consumption in demand  $j$  at time  $t$ . The state variable  $Q_i(t)$  is the residual stock of resource  $i$  over time. The two terms in (1) denote the standard sum of consumer plus producer surplus. The current value Hamiltonian for the above problem is given by

$$H = \left[ \sum_{j \in U} \int_0^Z D_j^{-1}(x) dx - \sum_{i \in R} \sum_{j \in U} w_{ij} q_{ij}(t) \right] dt - \sum_{i \in R} \lambda_i(t) \sum_{j \in U} q_{ij}(t),$$

where  $\lambda_i(t) \geq 0$  has the standard interpretation as the scarcity rent of resource  $i$ . The solution is defined in terms of optimal price paths as functions of time. Let the price of the resource input for demand  $j$  be  $p_j(t) \equiv D_j^{-1}(\sum_{i \in R} q_{ij}(t))$ . The necessary conditions for a solution are:

$$\dot{Q}_i(t) = - \sum_{j \in U} q_{ij}(t) \quad \text{for } i \in R, \tag{4}$$

$$\dot{\lambda}_i(t) = r \lambda_i(t) \quad \text{for } i \in R, \tag{5}$$

$$p_j(t) = w_{ij} + \lambda_i(t) \text{ (if } < \text{ then } q_{ij}(t) = 0) \quad \text{for } i \in R \text{ and } j \in U, \tag{6}$$

and

$$\lim_{t \rightarrow \infty} e^{-rt} \lambda_i(t) Q_i(t) = 0 \quad \text{for } i \in R. \tag{7}$$

It is straightforward to show that a solution to the above program exists:

**Proposition 1.** *There exists a unique optimal solution to program (1)–(3) and the necessary conditions (4)–(7) are also sufficient.*

**Proof.** See Appendix 1.  $\square$

Before proceeding, we prove the intuitive but useful result that all resources approach exhaustion in the limit.

**Lemma 1.**  $\lim_{t \rightarrow \infty} Q_i(t) = 0$  for  $i \in R$ .

**Proof.** Pick  $a \in R$  and suppose that  $\lambda_a(t_0) = 0$ . From (5),  $\lambda_a(t) = 0$  and so from (6),  $p_j(t) \leq w_{aj}$ . Since demand is positive and downward sloping, for some  $j \in U$ ,  $0 < D_j(w_{aj}) \leq D_j(p_j(t)) = \sum_{i=1}^m q_{ij}(t)$ . Thus  $\int_{t_0}^{\infty} \sum_{i=1}^m q_{ij}(t) dt = \infty$  so there exists  $b \in R$  such that  $\int_{t_0}^{\infty} q_{bj}(t) dt = \infty$ . From (4),  $Q_b(t) = -\sum_{j \in U} q_{bj}(t) \leq -q_{bj}(t)$  and so eventually  $Q_b(t)$  will become negative which contradicts (2). Thus the supposition is false and so  $\lambda_a(t_0) > 0$ . Combining (5) and (7) yields  $0 = \lim_{t \rightarrow \infty} e^{-rt} \lambda_a(t) Q_a(t) = \lim_{t \rightarrow \infty} e^{-rt} \lambda_a(t_0) e^{rt} Q_a(t) = \lambda_a(t_0) \lim_{t \rightarrow \infty} Q_a(t)$  which since  $\lambda_a(t_0) > 0$  implies that  $\lim_{t \rightarrow \infty} Q_a(t) = 0$ . Since  $a$  was arbitrary, then  $\lim_{t \rightarrow \infty} Q_i(t) = 0$  for  $i \in R$ .  $\square$

### 2.1. Hotelling scarcity rents in the heterogenous demand framework

The necessary conditions (4)–(7) can be easily interpreted. Condition (4) is just a restatement of (3). Condition (5) is the familiar Hotelling equation which suggests that scarcity rents rise over time at the rate of discount. Condition (6) is the basic Kuhn–Tucker condition governing resource allocation in the multiple demand model that says that the price in any given demand cannot exceed the net cost of any resource in that demand. This inequality implies that the resource that is available at the lowest price (net cost plus scarcity rent) is always used for each demand, as proved by the following proposition:

**Proposition 2.** *The price (net cost plus scarcity rent) of a resource that is supplied for a given demand is no more than that of any alternative resource.*

**Proof.** Suppose that  $q_{aj}(t) > 0$  for some  $a \in R$ ,  $j \in U$  and  $t \in (t_0, \infty)$ . From (6)  $w_{aj} + \lambda_a(t) = p_j(t) \leq w_{ij} + \lambda_i(t)$  for  $i \in R$ .  $\square$

Solving (5) produces the familiar Hotelling equation

$$\dot{\lambda}_i(t) = \lambda_i(t_0)e^{rt} \quad \text{for } i \in R, \tag{8}$$

which states that the scarcity rent rises at the rate of discount. Condition (8) also implies that the scarcity rents of all resources are ordered. Based on this ordering, we write  $\lambda_a < \lambda_b$  to mean  $\lambda_a(t) < \lambda_b(t)$  for all  $t \in [t_0, \infty)$ . It may also be the case that the scarcity rents of two resources are the same. As shown by the following proposition, this must be the case if two resources ever simultaneously supply the same demand.

**Proposition 3.** *Two resources simultaneously supplying the same demand have the same scarcity rent and net cost for that demand.*

**Proof.** Let  $q_{aj}(t) > 0$  and  $q_{bj}(t) > 0$  for some  $a, b \in R$ ,  $j \in U$ , and  $t \in I$  where  $I \subset (t_0, \infty)$  is an open interval. From (6),

$$w_{aj} + \lambda_a(t) = p_j(t) = w_{bj} + \lambda_b(t) \quad \text{for } t \in I. \tag{9}$$

Differentiating, we get  $\dot{\lambda}_a(t) = \dot{\lambda}_b(t)$  for  $t \in I$ . From (5) and (8),  $\lambda_a = \lambda_b$ . That is, the scarcity rents are the same. Combining with (9) yields  $w_{aj} = w_{bj}$ . That is, the net costs are equal.  $\square$

A corollary to the above result is:

**Proposition 4.** *The prices of two demands that are simultaneously supplied by the same resource must grow at the same rate.*

**Proof.** Let resource  $i$  supply both demands  $j$  and  $k$  over an interval  $I \subset (t_0, \infty)$ . Then  $p_j(t) = w_{ij} + \lambda_i(t), p_k(t) = w_{ik} + \lambda_i(t)$  which implies that  $\dot{p}_j(t) = \dot{\lambda}_i(t) = \dot{p}_k(t) \forall t \in I$ .  $\square$

## 2.2. Ricardian absolute advantage

In the standard Hotelling/Herfindahl model with a single demand, resource rents are ordered by grade: the resource with the highest grade has the highest scarcity rent. With heterogenous demand, the ordering of resource rents is more problematic since there is not necessarily an ordering of costs among multiple resources. One resource may be cheaper for one demand and more costly for another demand when compared to other resources. The following definitions relate three different types of cost orderings that may occur:

**Definition.** Resource  $a \in R$  has an absolute advantage relative to resource  $b \in R$  in demand  $j \in U$  if  $w_{aj} < w_{bj}$ , some  $j \in U$ .

**Definition.** Resource  $a \in R$  dominates resource  $b \in R$ , if it has an absolute advantage in all demands, i.e., if  $w_{aj} < w_{bj}$ , all  $j \in U$ .

**Definition.** Resource  $a \in R$  is universally dominant if it dominates all other resources, i.e., if  $w_{aj} < w_{ij}$ , all  $i \in R, j \in U$ .

Absolute advantage implies lower net cost relative to another resource in a single demand. A resource that dominates another, has Ricardian absolute advantage relative to this other resource in *all* demands. A resource that universally dominates has absolute advantage, i.e., is strictly cheaper, relative to *all* resources and for *all* demands. The next results generalize the principle of cost-ordered scarcity rents.

**Proposition 5.** *Dominant resources have a higher scarcity rent.*

**Proof.** Let  $w_{aj} < w_{bj}$  for resources  $a, b \in R$  and all demands  $j \in U$ . Suppose that  $\lambda_a \leq \lambda_b$ . From (6),  $p_j(t) \leq w_{aj} + \lambda_a(t) < w_{bj} + \lambda_b(t)$  for all  $t \in (t_0, \infty)$ . Thus  $q_{bj}(t) = 0$  for all  $t \in (t_0, \infty)$  and  $j \in U$ ; resource  $b$  is never extracted for any demand. Since this contradicts the Lemma, the supposition is false and thus  $\lambda_a > \lambda_b$ .  $\square$

By the above proposition, a resource that *universally dominates* (if one exists) will have the highest scarcity rent, since its scarcity rent will be higher than that of all other resources. Note that it is not possible to order resources with only absolute advantage by their scarcity rents. This is because resource  $a$  ( $b$ ) may have *absolute advantage* over resource  $b$  ( $a$ ) in demand  $j$  ( $k$ ), so that no resource is *dominant*.

### 2.3. Resources and grades

The multiple-demand framework allows for a clear distinction between individual resources and grades of a single resource:

**Definition.** Resource  $a, b \in R$  are of the same resource class if  $w_{bj} - w_{aj} = d$  for all  $j \in U$  and some constant  $d$ . Furthermore, if  $d > 0 (d = 0, d < 0)$  then resource  $a$  is a higher grade (same grade, lower grade) of the resource class than resource  $b$ .

This formal definition of *resource class* corresponds roughly to what is meant in common parlance by distinguishing between resources of different types, e.g. ‘coal’, ‘oil’, ‘natural gas’, etc. The difference between resources within any class is only cost – a higher-grade resource has the same cost advantage regardless of demand. This classification is based on the economic properties of the resource, not its chemical properties. Two resources with similar chemical compositions, e.g. light vs. heavy crude oil, may be in different resource classes, depending on whether their cost advantage varies across uses. In the Herfindahl case of one demand, all resources are in the same resource class, i.e. there is no distinction between different resources and different grades of the same resource.

**Proposition 6.** *Higher-grade resources have a larger scarcity rent.*

**Proof.** If resource  $a \in R$  is a higher grade of the same resource class than resource  $b \in R$ , then by definition,  $w_{bj} - w_{aj} = d > 0$  for  $j \in U$  which implies that  $w_{aj} < w_{bj}$  for  $j \in U$ . From Proposition 5, resource  $a$  has a larger scarcity rent than resource  $b$ .  $\square$

That is, a higher grade of the same resource class *dominates* the grade to which it is being compared, and the highest grade *dominates* all other grades. With only one resource class but multiple demands, the highest grade is *universally dominant*. With homogenous (one) demand, the Herfindahl principle states that resources are extracted sequentially, in order of cost. The following two propositions generalize this principle to show that the use of resources within each demand is always in order of *absolute advantage* and that resources are extracted by decreasing grade.

**Proposition 7.** *Resources are supplied for a given demand in order of absolute advantage.*

**Proof.** We show that if a resource is supplied for a given demand then a lower net cost resource will not subsequently be supplied for that demand. Thus by definition, resources supplied for a given demand must be in order of absolute advantage. Let  $q_{aj}(t_1) > 0$  and  $w_{aj} > w_{bj}$  for resources  $a, b \in R$  and demand  $j \in U$  at time  $t_1 \in (t_0, \infty)$ . From (6),

$$w_{aj} + \lambda_a(t_1) = p_j(t_1) \leq w_{bj} + \lambda_b(t_1), \tag{10}$$

which since  $w_{aj} > w_{bj}$  implies that  $\lambda_a < \lambda_b$ . From (5),  $\dot{\lambda}_a < \dot{\lambda}_b$  and so the left-hand side of (10) increases at a slower rate than the right-hand side. Thus  $w_{aj} + \lambda_a(t) < w_{bj} + \lambda_b(t)$  for all  $t \in (t_1, \infty)$ . Then from (6),  $p_j(t) \leq \lambda_a(t) + w_{aj} < \lambda_b(t) + w_{bj}$  for all  $t \in (t_1, \infty)$  and so  $q_{bj}(t) = 0$  for all  $t \in (t_1, \infty)$ .  $\square$

Proposition 7 does not say that all resources will be supplied for each demand but that of those resources that are supplied, their use will be in strict order of absolute advantage. In this sense, the Herfindahl Principle of ‘least-cost-first’ is preserved within each demand. In the special case of a single demand, Proposition 7 reduces to the Herfindahl principle.

**Proposition 8.** *Resources of the same resource class are extracted in order of decreasing grade.*

**Proof.** We show that if one resource is being extracted, then a higher grade of the same resource will not subsequently be extracted. Thus resources within the same class are extracted in order of decreasing grade. Let  $\dot{Q}_a(t_1) > 0$  and  $\dot{Q}_b(t_1) = 0$  for resources  $a, b \in R$  at time  $t_1 \in (t_0, \infty)$  where resource  $b$  is a higher grade of the same resource class as resource  $a$ . By the last inequality, from (4) there exists  $c \in U$  such that  $q_{ac}(t_1) > 0$ . From (6),

$$w_{ac} + \lambda_a(t_1) = p_c(t_1) \leq w_{bc} + \lambda_b(t_1), \tag{11}$$

which since  $w_{bc} < w_{ac}$  from the definition of higher grade, implies that  $\lambda_a(t_1) < \lambda_b(t_1)$ . From (5),  $\dot{\lambda}_a < \dot{\lambda}_b$ , the left-hand side of (11) increases at a slower rate than the right-hand side, and so  $w_{ac} + \lambda_a(t) < w_{bc} + \lambda_b(t)$  for all  $t \in (t_1, \infty)$ . Since  $w_{bj} - w_{aj}$  is constant for all  $j \in U$  (from the definition of resource class), this implies that  $w_{aj} + \lambda_a(t) < w_{bj} + \lambda_b(t)$  for all  $t \in (t_1, \infty)$  and  $j \in U$ . Combining with (6) yields  $p_j(t) \leq w_{aj} + \lambda_a(t) < w_{bj} + \lambda_b(t)$  which implies that  $q_{bj}(t) = 0$  for all  $t \in (t_1, \infty)$  and  $j \in U$ . From (4),  $\dot{Q}_b(t) = -\sum_{j \in U} q_{bj}(t) = 0$  for all  $t \in (t_1, \infty)$ .  $\square$

If there is a single resource class, all demands can be aggregated into one composite demand and Proposition 8 reduces to the Herfindahl Principle. Since the proposition demonstrates that deposits within a resource class will be extracted in strict order of grade, we can aggregate resource grades and consider the resulting composite resource as having an extraction cost function that increases with cumulative extraction. This provides a microeconomic foundation for resources with rising, cumulative extraction cost functions, used frequently in the literature (e.g. Heal, 1976).

Since demand is positive at all prices, there will always be a resource available for each demand. The following provides a condition under which all resources except one will be exhausted.

**Proposition 9.** *A resource that is strictly inferior, i.e. dominated by all other resources, will eventually be used exclusively for all demands.*

**Proof.** Let resource  $a \in R$  be strictly inferior. From Proposition 5,  $\lambda_a < \lambda_i$  for all  $i \in R - \{a\}$ . Since scarcity rents rise exponentially, there exists a time  $t_a \in (t_0, \infty)$  such that  $w_{aj} + \lambda_a(t) < w_{ij} + \lambda_i(t)$  for all  $i \in R - \{a\}$ ,  $j \in U$ ,  $t \in (t_a, \infty)$ . From (6),  $q_{ij}(t) = 0$  for all  $i \in R - \{a\}$ ,  $t \in (t_a, \infty)$  and so  $q_{aj}(t) > 0$  for  $j \in U$ ,  $t \in (t_a, \infty)$ .  $\square$

Given that scarcity rents are ordered, a strictly inferior resource will have the lowest scarcity rent in all stages. A strictly inferior resource that is unlimited in quantity thus corresponds to the notion of ‘backstop resource’ in the resource

economics literature. At the other end of the spectrum, it is natural to ask if a single resource could be used exclusively for all demands at time  $t_0$ . The next proposition demonstrates that if there is a resource that is relatively cheap and plentiful, it will be used exclusively, i.e., for all demands at the beginning of the extraction program.<sup>8</sup>

**Proposition 10.** *A resource  $a \in R$  that is universally dominant will be used exclusively at time  $t_0$  if*

$$\sum_{j \in U} \int_{s_0 + w_{aj}}^{s_j + w_{aj}} D_j(p)/(p - w_{aj}) dp < rQ_a(t_0)$$

where

$$s_0 = \min\{w_{ij} - w_{aj} \mid i \in R - \{a\}, j \in U\},$$

and

$$s_j = \max\{w_{ij} - w_{aj} \mid i \in R, j \in U\}.$$

**Proof.** See Appendix 1.  $\square$

On the other hand, if each resource has absolute advantage in one demand, then no resource can supply all demands in any stage. This is shown below for the case of an equal number of resources and demands (i.e.,  $m = n$ ), sometimes referred to as ‘even’ models.<sup>9</sup>

**Proposition 11.** *When each resource has absolute advantage in an ‘even’ model, no resource can supply all demands.*

**Proof.** Without loss of generality, re-label all demands from  $1, \dots, m$  so that resource  $i$  has absolute advantage in demand  $i$ ,  $i = 1, \dots, m$ , that is,  $w_{ii} < w_{ji}, \forall j \neq i$ . Let resource 1 be used for all demands, for some  $t \in I$  where  $I \subset (t_0, \infty)$ . Then define  $\phi_{1j}^j(t) = p_{1j}(t) - p_{jj}(t)$ ,  $j \in \{U \setminus 1\}$ . That is,  $\phi_{1j}^j$  is the price differential between resource 1 and resource  $j$  in demand  $j$ . Since resource 1 is used for all demands in this interval,  $\phi_{1j}^j(t) = (\lambda_1(t) - \lambda_j(t)) + (w_{1j} - w_{jj}) < 0$ . This implies that  $\lambda_1(t) < \lambda_j(t) \forall j \neq 1$  since  $w_{1j} > w_{jj}$ . By (5), this inequality holds over the entire time path. Also, resource 1 is used for all demands, so it is cheaper relative to any other resource  $k$  in demand  $j$ , i.e.,  $\phi_{1k}^j(t) < 0 \forall t \in I$ . Since resource 1 has the lowest shadow price,  $\dot{\phi}_{1k}^j(t) = \dot{\lambda}_1(t) - \dot{\lambda}_k(t) = r(\lambda_1(t) - \lambda_k(t)) < 0 \forall t \in [t_0, \infty), k \in R, j \in \{U \setminus 1\}$ . That is, resource 1 will always be cheaper than other resources for every demand. No other resource will ever be used subsequently, contradicting Lemma 1.  $\square$

The above proposition is independent of the stock sizes of the resources. That is, however *abundant* a resource may be, it will never be the exclusive supplier for all

<sup>8</sup>An intuitive explanation for this result is provided for the  $2 \times 2$  case in Section 3.

<sup>9</sup>The even-odd terminology follows extensions of Heckscher–Ohlin trade theory to higher dimensions. For an overview, see Bhagwati et al. (1998).

demands. As resources get exhausted, one resource may become universally dominant and thus become the exclusive supplier.

#### 2.4. Ricardian comparative advantage

In the following, we develop notions of comparative advantage and show that this taxonomy plays an important part in characterizing the sequence of resource extraction. In particular, a resource with universal dominance and universal comparative advantage will *always* be used first. We thus generalize the Herfindahl notion to the case with multiple demands.

**Definition.** Resource  $a \in R$  has a pairwise comparative advantage in use  $k$  over resource  $b \in R$  if  $w_{bk} - w_{ak} > w_{bj} - w_{aj}$ , some  $j \in \{U \setminus k\}$ .<sup>10</sup>

**Definition.** Resource  $a \in R$  has comparative advantage in use  $k$  over resource  $b \in R$  if  $w_{bk} - w_{ak} > w_{bj} - w_{aj}$ , all  $j \in \{U \setminus k\}$ .<sup>11</sup>

**Definition.** Resource  $a \in R$  has universal comparative advantage in use  $k$  if it has a comparative advantage over all other resources, i.e. if  $w_{ik} - w_{ak} > w_{ij} - w_{aj}$ , all  $j \in \{U \setminus k\}$ ,  $i \in \{R \setminus a\}$ .

These definitions parallel the notions of absolute advantage presented earlier. If oil has pairwise *comparative advantage* relative to coal in transportation, then the net cost differential between oil and coal in transportation exceeds the net cost differential in *some* other demand.<sup>12</sup> *Comparative advantage* of oil in transportation, say relative to coal implies that the net cost differential between oil and coal is higher in transportation than in *every* other demand. Comparative advantage over another resource implies pairwise *comparative advantage* over all demands. *Universal comparative advantage* of oil in transportation implies that the net cost differential between oil and *all* other resources is higher in transportation than in *every* other demand. Universal comparative advantage in a given use implies comparative advantage relative to all other resources. This taxonomy allows us to develop criteria for ranking resources by their comparative advantage.

The following result specifies that if two resources have pairwise comparative advantage, they cannot simultaneously be extracted for the uses wherein the *other*

<sup>10</sup>The last inequality implies that  $w_{aj} - w_{bj} < w_{ak} - w_{bk}$ . That is, the definition of pairwise comparative advantage is *symmetric*, i.e., resource  $b$  has a pairwise comparative advantage in use  $j$  over resource  $a$ . However, it is easy to check that it is also *transitive*, i.e., if resource  $a$  has pairwise ( $k$  and  $j$ ) comparative advantage in use  $k$  relative to resource  $b$ , and  $b$  has pairwise ( $k$  and  $j$ ) comparative advantage in use  $k$  relative to resource  $c$ , then resource  $a$  has pairwise ( $k$  and  $j$ ) comparative advantage in use  $k$  relative to resource  $c$ .

<sup>11</sup>Similar distinctions between the three types of comparative advantage may be useful in international trade theory as well.

<sup>12</sup>If distinct resources have absolute advantage in specific demands, it leads to pairwise comparative advantage. Suppose resource  $a(b)$  has absolute advantage over resource  $b(a)$  in demand  $j(k)$ . Then  $w_{aj} < w_{bj}$  and  $w_{bk} < w_{ak}$  which upon subtracting inequalities yields  $w_{bk} - w_{ak} < w_{bj} - w_{aj}$ , implying that resource  $a(b)$  has pairwise comparative advantage relative to resource  $b(a)$  in use  $j(k)$ . From Proposition 12 below, no resource can be an exclusive supplier under pairwise comparative advantage.

resource has the advantage. This result limits the set of solutions, as we will see in the  $2 \times 2$  case.

**Proposition 12.** *If resources  $a$  and  $b$  have pairwise comparative advantage relative to one another in  $j$  and  $k$ , respectively, then  $a$  cannot be used in  $k$  while  $b$  is used in  $j$ ;  $a, b \in R$ ;  $j, k \in U$ .*

**Proof.** Let  $q_{ak}(t) > 0$ ,  $q_{bj}(t) > 0$  for  $t \in I$  where  $I \subset (t_0, \infty)$ . By the definition of pairwise comparative advantage,  $w_{bk} - w_{bj} < w_{ak} - w_{aj}$ . Then from Proposition 2,  $p_k(t) = w_{ak} + \lambda_a(t) \leq w_{bk} + \lambda_b$  and  $p_j(t) = w_{bj} + \lambda_b(t) \leq w_{aj} + \lambda_a(t)$ . Subtracting the inequalities yields  $\lambda_b + w_{bk} - (\lambda_b + w_{bj}) \geq \lambda_a + w_{ak} - (\lambda_a + w_{aj})$  so that  $w_{bk} - w_{bj} \geq w_{ak} - w_{aj}$  which is a contradiction.  $\square$

Next we show that both universal dominance and universal comparative advantage are sufficient to ensure that a resource is used before any other resource:

**Proposition 13.** *A resource with universal dominance and universal comparative advantage in demand  $k$  must be used exclusively in that demand.*<sup>13</sup>

**Proof.** Let resource  $a$  have universal dominance and universal comparative advantage in use  $k$ . Let another resource  $b \in R$  supply demand  $k$  over an interval  $I_1 = [t_1, t_2] \subset [t_0, \infty)$ . Then  $w_{ak} + \lambda_a(t) = p_k(t) \geq w_{bk} + \lambda_b(t) \forall t \in I_1$  so that  $0 < w_{bk} - w_{ak} \leq \lambda_a(t) - \lambda_b(t)$ , where the first inequality is from universal dominance of  $a$  in  $k$ . However, over another interval  $I_2 = [t_3, t_4] \subset (t_2, \infty)$ , resource  $a$  must be used for some demand, say  $j$ . Then  $p_j(t) = w_{aj} + \lambda_a(t) \leq w_{bj} + \lambda_b(t) \forall t \in I_2$  which implies that  $\lambda_a(t) - \lambda_b(t) \leq w_{bj} - w_{aj} \forall t \in I_2$ . Since  $\lambda_a(t) \geq \lambda_b(t)$  from above, the shadow prices must diverge by (5), hence  $\dot{\lambda}_a(t) - \dot{\lambda}_b(t) \geq 0$  and by the definition of the intervals  $I_1$  and  $I_2$ ,  $\{\lambda_a(t) - \lambda_b(t) \mid t \in I_1\} \leq \{\lambda_a(t) - \lambda_b(t) \mid t \in I_2\}$ . Consolidating, we get  $w_{bk} - w_{ak} \leq \{\lambda_a(t) - \lambda_b(t) \mid t \in I_1\} \leq \{\lambda_a(t) - \lambda_b(t) \mid t \in I_2\} \leq w_{bj} - w_{aj}$ . This inequality must hold for any resource  $b$  and demand  $j$ , i.e.,  $w_{ik} - w_{ak} \leq w_{ij} - w_{aj}$ , which contradicts the definition of universal comparative advantage of resource  $a$  in use  $k$ .  $\square$

A resource with universal dominance and universal comparative advantage in demand  $k$  must be used exclusively in that demand at the beginning of the planning horizon. Universal dominance and universal comparative advantage are sufficient for the above result to hold. They establish a clear ordering of resource extraction across resource classes.

The two definitions of absolute advantage – dominance and universal dominance – are equivalent when there are only two resources. In this case, all three definitions of comparative advantage are also equivalent. This is because, with two demands, if

<sup>13</sup>Our notion of pair-wise comparative advantage is equivalent to the definition of comparative advantage put forward by Gaudet et al. (2001). Comparative advantage, in their model determines, given an arbitrary use profile, which city will switch *first* to a higher cost landfill site. A city (in our case, demand) may switch to a more costly site (in our analogy, use a more costly resource) if it has comparative advantage in that demand. However, their definition of comparative advantage focuses on resource switching and does not predict which resource will be used *first*. Our (stronger) definition of comparative advantage suggests that if a resource with universal dominance and universal comparative advantage exists, it is automatically picked as the exclusive supplier at the beginning of the planning horizon.

resource  $a$  has dominant comparative advantage in use  $k$  relative to resource  $b$ , the cost differential between resource  $a$  and  $b$  in demand  $k$  is higher than in all *other* demands. If the number of demands is only two, there is only one ‘other’ demand. Thus  $a$  has universal comparative advantage over  $b$  in use  $k$ . The argument for the other equivalencies is similar.<sup>14</sup> In the Herfindahl case when the number of demands is unity, the definitions of comparative advantage reduce to the following:

**Definition.** Under one demand, resource  $a$  has universal comparative advantage if  $w_{ik} - w_{ak} > 0$ ,  $i \in \{R \setminus a\}$ .

That is, a resource has comparative advantage when its net cost is lower than that of *all other* resources. Then Proposition 13 reduces to the Herfindahl Principle, i.e., the resource with the *lowest net cost* must be used exclusively. Furthermore, the sequence of extraction must be according to the ‘least-cost-first’ principle. Unlike in the one demand case (except when resources have equal net costs), a resource with universal comparative advantage may not exist under multiple demands.

### 2.5. Absolute and comparative advantage: polar cases

The role of absolute and comparative advantage in the general case may be illuminated by two polar extremes: (i) for each resource, conversion costs are the same across end uses (ii) each resource enjoys a symmetrical comparative advantage in each end use.

For (i), with equal conversion costs across demands for each resource, we have  $v_{ij} = v_{ik} \forall j, k \in R, i \in U$ . Then  $w_{ij} = c_i + v_{ij} = c_i + v_{ik} = w_{ik}$ , so that for any two resources  $a$  and  $b$ ,  $w_{bj} - w_{aj} = c_b + v_{bj} - (c_a + v_{aj}) = c_b + v_{bk} - (c_a + v_{ak}) = w_{bk} - w_{ak} \forall j, k \in U$ , since  $v_{ij} = v_{ik}$ . Hence by definition, all resources are of the same resource class. Let us re-label the resources in order of increasing net cost, i.e.,  $w_{1j} < w_{2j} < \dots < w_{mj}$ . Then by Proposition 6,  $\lambda_1 > \lambda_2 > \dots > \lambda_m$ . By Proposition 8, resources will be extracted in order of decreasing ‘grade’, with resource 1 in the initial stage and  $m$  at the end. The sequence of extraction will follow the Herfindahl Principle, i.e., each resource will supply all demands exclusively, until it is exhausted and the next higher net cost resource is employed.

For (ii), consider the  $m \times m$  model, in which each resource has universal comparative advantage in a given demand. Suppose demands are identical and stocks of each resource are equal. Without loss of generality, the demands are re-labeled such that resource  $i$  has universal comparative advantage in demand  $i$ ,  $i \in I = \{1, \dots, m\}$ . Suppose  $w_{11} = w_{22} = \dots = w_{mm} < w_{1i_1} = w_{2i_2} = \dots = w_{mi_m}$ , where  $i_1 \in \{I \setminus 1\}$ ,  $i_2 \in \{I \setminus 2\}, \dots, i_m \in \{I \setminus m\}$ . That is, the net costs of all resources are equal in their respective demands and higher (and equal) for all other demands. In this perfectly symmetrical world, since there is no distinction between resources except by comparative advantage, and demands are identical,  $\lambda_1 = \lambda_2 = \dots = \lambda_m$ . By

<sup>14</sup>In any two-resource model, comparative advantage and universal comparative advantage are equivalent. All three definitions are distinct in models with dimensionality  $m \geq 2, n > 2$ . Precise results on their relationships in different dimensions can be explored in future work.

Proposition 2, each resource will be used exclusively in the demand in which it has comparative advantage. Because of perfect symmetry, each resource will be exhausted at infinity. To summarize, in the first polar extreme, resource use is determined entirely by absolute advantage; in the second, entirely by comparative advantage. More generally, resource use will reflect a trade-off between both forces.

### 2.6. Comparative dynamics

In order to investigate the comparative dynamics properties of the  $m \times n$  model, we can define the value function in (1)–(3) as

$$V(Q_i(t_0)) = \int_{t_0}^{\infty} B(Q_i(t_0), t) dt, \tag{12}$$

where

$$B(Q_i(t_0), t) = e^{-rt} \left[ \sum_{j \in U} \int_0^Z D_j^{-1}(x) dx - \sum_{i \in R} \sum_{j \in U} w_{ij} q_{ij}(t) \right]$$

represents the discounted benefits from extraction at any given instant of time  $t$ . To relate the change in resource scarcity rents to the change in the aggregate resource stocks, we need to establish the differentiability of the value function  $V(Q_i(t_0), t)$ . Benveniste and Scheinkman (1979) have shown that under fairly general conditions as in our case, the value function  $V(\cdot)$  is once differentiable. In particular, we invoke their Corollary 1, where the optimal control is a piecewise continuous function of time. In our case, the control functions are discontinuous since there may exist intervals  $I$  such that  $q_{ij}(t) = 0$ ,  $t \in I$  and  $I \subset (t_0, \infty)$ . By Proposition 7, and because we have only a finite number of demands and resources, there can only be a finite number of such intervals. At the switch points between these intervals, the state variables may not be differentiable and the control functions may be discontinuous. In Appendix B, we check that (12) satisfies the assumptions of their Corollary 1. This gives the following result:

$$\partial V(Q_i(t_0, t)) / \partial Q_i(t_0) = \lambda_i(t_0),$$

which implies that the initial scarcity rent is the derivative of the optimal value function with respect to the initial stock of the resource. In another paper, under more general conditions, Benveniste and Scheinkman (1982) have shown that the value function is concave. That is, if  $Q_i(t_0), \hat{Q}_i(t_0)$  are two different initial stocks of any resource  $i$ , then  $(Q_i(t_0) - \hat{Q}_i(t_0)) \cdot (\lambda_i(t_0) - \hat{\lambda}_i(t_0)) \leq 0$ , where  $\lambda_i(t_0), \hat{\lambda}_i(t_0)$  represent the corresponding vector of scarcity rents at time  $t_0$ . Thus if  $\hat{Q}_i(t_0) > Q_i(t_0)$ , e.g., representing an exogenous increase in the initial stock of resource  $i$ , then  $\hat{\lambda}_i(t_0) \geq \lambda_i(t_0)$ , provided that all other stocks remain unchanged. We can then state the following result:

**Proposition 14.** *An exogenous increase in the initial stock of a resource causes a reduction in its scarcity rent.*

This implies that in the limit, if a given resource  $i$  is inexhaustible,  $\lim_{Q_i(t_0) \rightarrow \infty} \lambda_i(t) = 0$  which yields  $\lim_{Q_i(t_0) \rightarrow \infty} P_{ij}(t) = w_{ij} + \lim_{Q_i(t_0) \rightarrow \infty} \lambda_i = w_{ij}$ . If all other resources are limited in quantity, the  $i$ th resource is a backstop resource. In Section 3, we examine the role of resource abundance for the  $2 \times 2$  case.

### 3. Characterization of the $2 \times 2$ case

We now consider the simplest possible setting with multiple demands, that of two demands and two resources. In this case, dominance implies universal dominance and pairwise comparative advantage is equivalent to comparative and universal comparative advantage. Without loss of generality, we assume that the demands are denoted by electricity and transportation, and the resources are oil and coal. There are only two possible cases to consider: (i) one resource is dominant, i.e., has absolute advantage in both demands and (ii) when each resource has absolute advantage, i.e., has absolute advantage in some demand. Again without loss of generality, we consider the following two cases: (i) oil is a dominant resource, and (ii) oil has absolute advantage in transportation and coal in electricity.<sup>15</sup>

#### 3.1. Oil is dominant

We can simplify notation by letting  $w_{OE}$  and  $w_{OT}$  ( $w_{CE}$  and  $w_{CT}$ ) denote net costs of oil (coal) to electricity and transportation, respectively. Note that dominance for oil implies that  $w_{OE} < w_{CE}$  and  $w_{OT} < w_{CT}$ . Define  $k = (w_{CT} - w_{OT}) + (w_{OE} - w_{CE})$ . Then oil has *comparative advantage* in transportation (electricity) if  $k > (<) 0$ . The following propositions provide sufficient conditions for oil to be used at the beginning for both uses, when it is dominant:

**Proposition 15.** *If oil is dominant and has comparative advantage in transportation, and the condition*

$$\int_{w_{CT-k}}^{w_{CT}} \frac{D_T(p)}{(p(t) - w_{OT})} dp < rQ_O(t_0)$$

*holds, then oil must be used for both uses at the beginning. In the second stage, oil is used for transportation and coal for electricity. In the third stage, coal is used for both uses.*

**Proof.** See Appendix 1.  $\square$

Proposition 15 suggests that only oil will be used at the beginning if it is abundant, the discount rate is high, the net cost of coal in transportation is high (since demand is downward sloping), the net cost of oil in transportation is low, the demand for transportation is low, and  $k$  is low. Recall that the magnitude of  $k$  denotes the degree

<sup>15</sup>A similar example is used in Chakravorty and Krulce (1994), albeit under the stricter assumption that oil is the lower cost resource for both demands and has a comparative advantage in transportation.

of comparative advantage of oil in transportation over coal in electricity. The higher the value of  $k$ , the less likely it is that oil will be used for electricity in the beginning stage. That is, comparative advantage of oil in transportation makes it less likely that oil will supply both demands in the initial stage. Note that in this stage, oil is used for electricity even though coal has comparative advantage in that demand. The effect of dominance is stronger than that of comparative advantage. The solution is shown in Fig. 1. Only oil is used until time  $t_1$ , followed by coal in electricity and oil in transportation until time  $t_2$ , when oil is exhausted. In the third stage, the strictly inferior resource, coal supplies both demands.

However, even if oil is dominant, i.e., has absolute advantage over coal in both demands, it may have comparative advantage in electricity, not in transportation. The following result shows that stage 2 will then be different. It is the mirror image of Proposition 15, so the proof is not given separately.

**Proposition 16.** *If oil is dominant and has comparative advantage in electricity, and the condition*

$$\int_{w_{CE-k}}^{w_{CE}} \frac{D_E(p)}{(p(t) - w_{OE})} dp < rQ_O(t_0)$$

*holds, then oil must be used for both uses at the beginning. In the second stage, oil is used for electricity and coal for transportation. In the third stage, coal is used for both uses.*

In this case  $k = (w_{CT} - w_{OT}) + (w_{OE} - w_{CE}) < 0$ . The solution is exactly as in Fig. 1 except that in stage 2, it is oil that is now used for electricity and

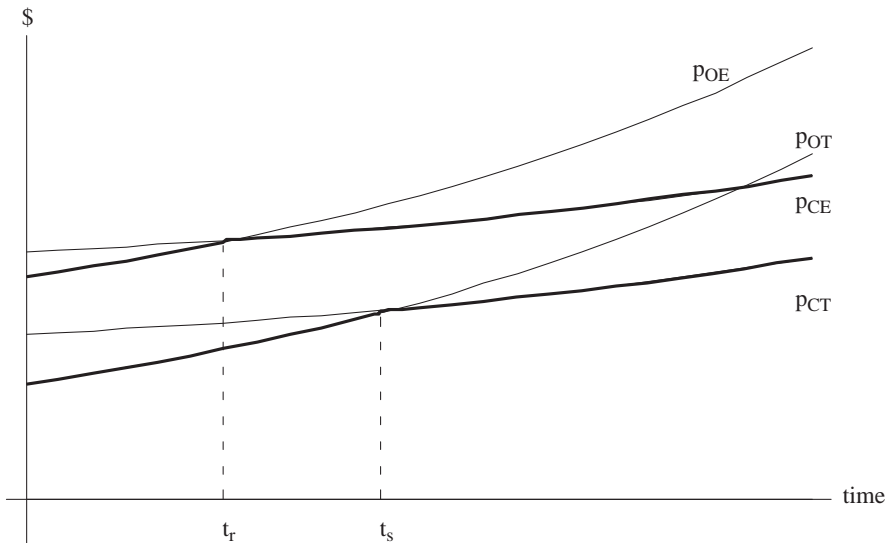


Fig. 1. Extraction profile when oil is dominant and has comparative advantage in transportation. Oil is used exclusively at the beginning and coal at the end.

Table 2  
Solutions to the two-resource two-demand case

E	T
(a) Oil has dominance and comparative advantage in transportation	
O	O
C	O
C	C
(b) Oil has dominance and comparative advantage in electricity	
O	O
O	C
C	C
(c) Oil has absolute advantage in transportation and coal in electricity; oil abundant	
C	O
O	O
(d) Oil has absolute advantage in transportation and coal in electricity; coal abundant	
C	O
C	C

Notes: E, electricity; T, transportation; O, oil; C, coal.  
Another set of four solutions can be generated by switching oil and coal.

coal in transportation. Both solutions are summarized in Table 2(a,b). A reversal is obtained in stage 2, and the sequence of extraction in this stage is strictly according to comparative advantage, as implied by Proposition 12. Since the solutions in Propositions 15 and 16 are unique, we get the following corollary:

**Corollary.** *When both resources are extracted simultaneously in a  $2 \times 2$  model, resources must be extracted for the demand in which they have comparative advantage.*

Since the stages of resource use are determined according to the principle of least-shadow-price-first, Fig. 2 shows the solution described in Proposition 15 in terms of shadow price differences. The functions  $\Phi_E(t)$  and  $\Phi_T(t)$  denote the price of oil net of coal in electricity and transportation, respectively.<sup>16</sup> When  $\Phi_E(t) < 0$ , the price of oil is lower than that of coal; hence oil is used for electricity. When  $\Phi_E(t) > 0$ , coal becomes cheaper and substitutes for oil. As shown in the proof of Proposition 15,  $\phi_E(t) > \phi_T(t)$ . When  $k = 0$ ,  $\phi_E(t) \equiv \phi_T(t)$  and the middle stage is eliminated. Oil is used exclusively at the beginning followed by coal at the end. The transition from oil to coal happens in both sectors simultaneously. Thus, the Herfindahl result of ‘least-cost-first’ is obtained as a special case when oil has absolute advantage in

<sup>16</sup>That is,  $\phi_E(t) = p_{OE}(t) - p_{CE}(t)$  and  $\phi_T(t) = p_{OT}(t) - p_{CT}(t)$ .

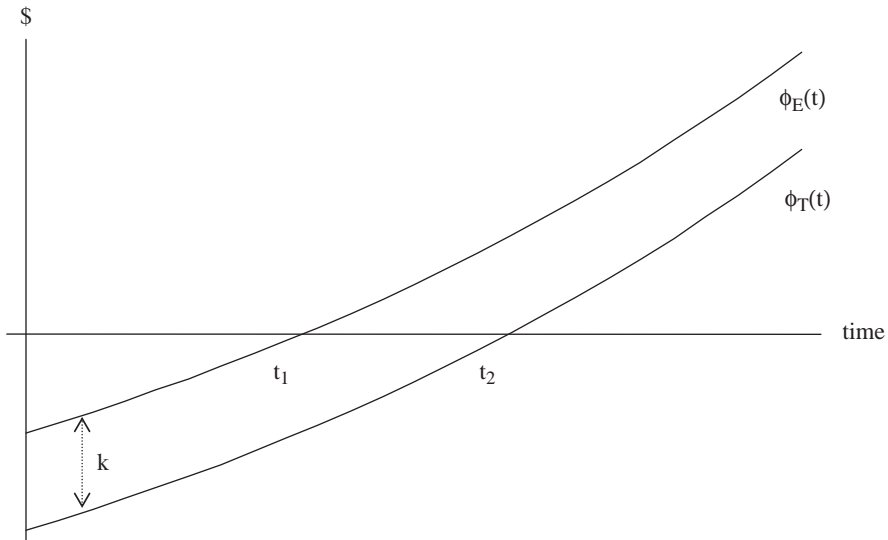


Fig. 2. Oil is dominant and has comparative advantage in transportation ( $k > 0$ ). Oil is used in electricity until  $t_1$  and in transportation until  $t_2$ .

transportation and coal in electricity, but no resource has comparative advantage. The cost advantage of oil and coal in their respective demands cancel each other.<sup>17</sup> The following proposition relates the length of the second stage to the magnitude of comparative advantage, denoted by  $k$ :

**Proposition 17.** *If oil is dominant and has comparative advantage in transportation, and the inequality stated in Proposition 15 holds, the length of the second stage increases with  $k$  and decreases with  $(w_{CE} - w_{OE})$  and  $r$ . It is given by*

$$t_2 - t_1 = \left[ \ln \left( 1 + \frac{k}{w_{CE} - w_{OE}} \right) \right] / r,$$

where  $t_1(t_2)$  is the switch point from stage 1(2) to stage 2(3).<sup>18</sup>

**Proof.** Since  $\varphi_E(t) = p_{OE}(t) - p_{CE}(t) = (\lambda_O(t_0) - \lambda_C(t_0))e^{rt} + (w_{OE} - w_{CE})$ ,  $\varphi_E(t_1) = (\lambda_O(t_0) - \lambda_C(t_0))e^{rt_1} + (w_{OE} - w_{CE}) = 0$ , so that

$$t_1 = \left[ \ln \frac{w_{CE} - w_{OE}}{\lambda_O(t_0) - \lambda_C(t_0)} \right] / r.$$

<sup>17</sup>The model of Chakravorty and Krulce (1994) is obtained by substituting  $w_{OE} = w_{OT} = c_O$ ,  $w_{CE} = c_C$ ,  $w_{CT} = c_C + z$ , so that  $k = w_{CT} - w_{CE} = z > 0$ . Their Proposition 1 is a special case of our Proposition 15.

<sup>18</sup>An analogous result could be obtained for the case of Proposition 16.

Similarly,  $\phi_T(t_2) = 0$  implies

$$t_2 = \left[ \ln \frac{w_{CT} - w_{OT}}{\lambda_O(t_0) - \lambda_C(t_0)} \right] / r.$$

Subtracting  $t_1$  from  $t_2$  and some algebraic manipulation yields the result.  $\square$

The above proposition suggests that the higher the magnitude of comparative advantage the longer the stage where both resources are extracted simultaneously. The higher the absolute advantage of oil in electricity, the smaller the length of the stage in which there is joint extraction of the two resources. This is intuitive because if oil has a high degree of absolute advantage in electricity, it is likely to be used in stage 1 and not in stage 2. This will decrease the use of oil for transportation in stage 2. Thus, dominance of oil leads it to be used in stage 1, while comparative advantage forces specialization of the two resources in stage 2. A higher discount rate decreases the length of the second stage. Since oil has absolute advantage in both resources, a higher discount rate implies that profits from the use of oil in transportation in stage 2 are discounted more heavily, reducing its duration. Conversely, as we saw in Proposition 15, a higher discount rate suggests that oil is better used in stage 1 for generating profits earlier in the time horizon.

### 3.2. Oil has absolute advantage in transportation and coal in electricity

We now consider the case wherein both resources have one absolute advantage each. We can then state the following:

**Proposition 18.** *When oil has absolute advantage in transportation and coal in electricity, there are only two possible solutions, each with two stages. The first stage is the same in both solutions – oil is used for transportation and coal for electricity. In the second stage, either oil or coal is used exclusively for both uses.*

**Proof.** Let  $\lambda_O(t_0) > \lambda_C(t_0)$ . Then from the proof of Proposition 9, coal will be used for both uses in the terminal stage. But  $\phi_E(t) = (\lambda_O(t) - \lambda_C(t)) + (w_{OE} - w_{CE}) > 0$  and is increasing in  $t$ . If  $\phi_T(t_0) = (\lambda_O(t_0) - \lambda_C(t_0)) + (w_{OT} - w_{CT}) > 0$  then since  $\dot{\phi}_T(t) > 0$ , oil will never be used, violating Lemma 1. Thus  $\phi_T(t_0) < 0$ . In the first stage, oil is used for transportation and coal for electricity, followed by coal in both uses. The proof for the case when  $\lambda_O(t_0) < \lambda_C(t_0)$  is similar and not repeated.<sup>19</sup>  $\square$

By Proposition 14, if coal is sufficiently ‘abundant’ relative to oil, its scarcity rent will fall. In that case,  $\lambda_C(t_0) < \lambda_O(t_0)$ , so that the terminal stage uses coal for both demands. This can be seen graphically in Fig. 2 with one modification:  $\phi_E(t)$  is now everywhere positive so that there is only one switch point from oil to coal in transportation. Similarly, abundance of oil will imply that oil will be the terminal resource. Both solutions are summarized in Table 2(c,d). The polar case is when the

<sup>19</sup>The measure zero event in which  $\lambda_O(t_0) = \lambda_C(t_0)$  yields a knife-edge equilibrium with complete specialization – oil is used for transportation and coal for electricity over the entire planning horizon. Both prices grow at the same rate. The price differentials  $\phi_E(t), \phi_T(t)$  are constant functions.

stock of oil is unlimited, and it becomes a (conventional) backstop resource. In that case, the scarcity rent of oil is zero. The price differentials  $\phi_E(t)$  and  $\phi_T(t)$  are decreasing in time  $t$ .

### 3.3. Clean oil, dirty coal: environmental taxes in the $2 \times 2$ case

The multiple demand framework allows for the analysis of taxes that are resource or sector-specific (or both). Again, for simplicity, consider oil to be a ‘clean’ resource and impose an exogenous tax on a ‘dirty’ resource (e.g., coal) which is assumed to generate significant negative externalities. Other cases in which a resource *and* sector-specific tax (say, on coal in electricity generation) or only a sector-specific tax (a tax on the transportation sector, for instance, to reduce local pollution) may be imposed are discussed briefly later in this section. The single demand framework is inadequate for this purpose because it does not allow fuel substitution across sectors. We can then state the following result where the term ‘abundance’ implies that the stock of oil is large enough to satisfy the inequality in Proposition 15:

**Proposition 19.** *If oil is abundant and has absolute advantage in transportation while coal has absolute advantage in electricity, a tax on coal equal to  $t_C > w_{OE} - w_{CE}$  will lead to oil being used exclusively in the beginning.*

**Proof.** With the above tax, the modified net cost of coal in electricity  $\tilde{w}_{CE}$  is defined as  $\tilde{w}_{CE} = w_{CE} + t_C > w_{CE} + w_{OE} - w_{CE} = w_{OE}$ . Oil has dominance. By hypothesis,  $k = (w_{CT} - w_{OT}) + (w_{OE} - w_{CE}) > 0$  and is unaffected by the tax. Proposition 15 then implies that oil will be used exclusively at the beginning.  $\square$

The important difference is that in the one-demand Hotelling model, a tax on the resource reduces its use. However, when oil has absolute advantage in transportation and coal in electricity, each resource is being extracted in the first stage (Proposition 18). If oil is sufficiently abundant, a large enough tax on coal will lead to a shutdown in coal extraction, and both demands will be exclusively supplied by oil for a period after which, both resources will again be extracted jointly. The tax allows for a postponement of coal consumption until the future. This policy may be beneficial if, for example, exogenous research and development (e.g., on clean coal technologies) over time reduces the negative environmental externalities from coal.<sup>20</sup>

If a sector-specific tax is imposed, e.g., on all fuels in the transportation sector, the outcome may depend on which of the two conditions (in Propositions 15 and 16) hold. One interesting case is when oil is a dominant and abundant<sup>21</sup> resource, and has comparative advantage in transportation. Consider stage 2 of Proposition 15, i.e., oil is being used in transportation and coal in electricity. A sufficiently large tax on the transportation sector will in effect reduce the magnitude of transportation

<sup>20</sup>The basic model could be extended to include differential environmental costs of fossil fuels. For instance, carbon emitted per unit of energy delivered from burning coal, oil and natural gas is approximately in the ratio 5:4:3. For analytical purposes, these costs could be modeled as extraction costs. The higher environmental damage from coal will be reflected in a higher net cost of coal in all demands.

<sup>21</sup>In the sense that the inequality in Proposition 15 holds.

demand, so that the inequality in Proposition 15 may now be satisfied, leading to a switch to stage 1, i.e., oil supplying both forms of energy. Coal will no longer be extracted until stage 2 is reached again in this perturbed model. That is, a tax on the *transportation* sector may induce a shift from coal to oil in *electricity*. After the tax, oil is extracted for both demands, so that aggregate consumption of oil may increase.

The implications of the tax may be quite different if the tax is resource *and* sector-specific, e.g., a tax on coal consumption in electricity.<sup>22</sup> In that case, a tax that is larger than  $k$  will result in oil having comparative advantage in electricity, and coal in transportation. Thus, before the tax, oil is used in transportation and coal in electricity. After the tax, oil is used in electricity and coal in transportation.<sup>23</sup> The tax causes a complete switching of fuels between the two sectors.

Relative to the predictions of the Hotelling model, taxing the resource or taxing the sector (or a combination) may have very different implications for the extraction profile and for achieving pollution targets, a potentially significant policy issue that needs to be considered in detail in future research. Although we do not have endogenous choice of conversion technology that converts resources into final demands (e.g., automobiles, power generation equipment), there may be differential impacts depending on the type of tax imposed. Without a formal model, however, it is not clear *ex ante* how, for example, the adoption of fuel-efficient automobiles will be affected by these alternative tax mechanisms.

### 3.4. Sectoral energy prices

The optimal resource use profile can be used to characterize sector-specific efficiency prices of energy.<sup>24</sup> Suppose again, that coal is ‘dirty’ and oil is ‘clean,’ and environmental costs of the resources are incorporated into the net cost so that  $w_{CT} > w_{CE} > w_{OE} > w_{OT}$ . This implies that oil has a comparative advantage in transportation, i.e.,  $k = (w_{CT} - w_{OT}) + (w_{OE} - w_{CE}) > 0$ .<sup>25</sup> Then we can state the following:

**Proposition 20.** *If oil is abundant, then the price of electricity is higher than that of transportation energy in the beginning and lower in the end. Within each stage, the sectoral energy prices grow at the same rate and the price differential is monotone decreasing over time.*

<sup>22</sup>The gasoline tax, popular in many countries, is essentially a resource and sector-specific tax.

<sup>23</sup>With this tax on coal in electricity, say  $t_{CE}$ ,  $k < t_{CE}$  implies  $(w_{CT} - w_{OT}) - (w_{CE} + t_{CE} - w_{OE}) = (w_{CT} - w_{OT}) - (\hat{w}_{CE} - w_{OE}) < 0$ . Now oil has comparative advantage in electricity and coal in transportation.

<sup>24</sup>More generally, the price of a resource that is efficiently employed in a sector plus its conversion cost in that sector can be thought of as the shadow price of an intermediate good required for production of the final good. If no intermediate good (such as energy) can be identified, the conversion cost can be taken as the non-resource production costs for the final good.

<sup>25</sup>The assumption of clean oil, dirty coal used here is stronger than that of oil with dominance and comparative advantage in transportation. The former provides an empirical justification for the latter more general set of assumptions.

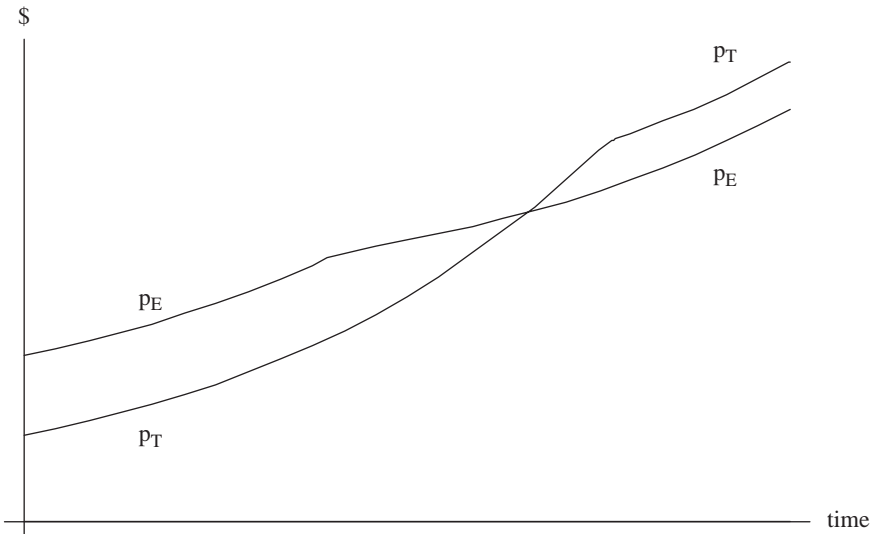


Fig. 3. Clean oil, dirty coal: the price path for transportation cuts the price of electricity from below.

**Proof.** By Proposition 15, the abundance of oil implies it is used exclusively at the beginning, and coal at the end. In the beginning stage,  $p_E - p_T = (\lambda_O + w_{OE}) - (\lambda_O + w_{OT}) = w_{OE} - w_{OT} > 0$ . In the terminal stage,  $p_E - p_T = (\lambda_C + w_{CE}) - (\lambda_C + w_{CT}) = w_{CE} - w_{CT} < 0$ . This proves the first part of the proposition. For the second part, note that  $p_E$  and  $p_T$  grow at the same rate in the first and last stages. In the middle stage,  $\dot{p}_T = \dot{\lambda}_O = r\lambda_O(t) > r\lambda_C(t) = \dot{\lambda}_C = \dot{p}_E$ . Thus  $p_E - p_T$  declines.  $\square$

The two price paths are shown in Fig. 3. In the first stage,  $p_E$  is higher than  $p_T$  and both grow at the same rate. In the second stage, the price of transportation energy rises at a faster rate than electricity, and cuts the latter from below. In the terminal stage, both prices again grow at an equal rate.<sup>26</sup> Since  $p_E - p_T$  declines over time, it follows that if the price of electricity is lower in the beginning then it must be lower for all subsequent periods. Notice that when oil has dominance and is used exclusively in the beginning, the inequality  $w_{OE} - w_{OT} > w_{CE} - w_{CT}$  holds, hence  $|p_E - p_T|$  is *always* lower in the terminal stage relative to the initial stage. Thus a general consequence of comparative advantage in the  $2 \times 2$  model is:

**Corollary.** *If a dominant resource is used at the beginning, the sectoral price differential must always decrease over time.*

<sup>26</sup>These sectoral price paths are purely a result of the relative ordering of net costs. The relative magnitude of demands in the two sectors plays a role only to the extent that the condition for Proposition 15 is satisfied. If the relative ordering of  $w_{OE}, w_{OT}$  ( $w_{CE}, w_{CT}$ ) were indeterminate, then the relative ordering of  $p_E, p_T$  would also be indeterminate. However, the price differential is driven by comparative advantage and still declines monotonically over time. It is easy to check that it holds even if there was no initial stage with exclusive use of oil.

#### 4. Concluding remarks

In a model of multiple resources that satisfy multiple demands, optimal resource use is determined by two forces. Specialization of resources according to demand is driven by Ricardian comparative advantage. The order of resource use over time is determined by Ricardian absolute advantage, whereby resources with the lowest extraction and conversion costs tend to be used first. Each principle partially masks the other, and only in polar cases do the pure forms of each principle emerge. In one such polar case, wherein conversion costs are independent of demand, resources will be used in order of absolute advantage, i.e. least cost first. At the other extreme, in a  $m \times m$  model, if each resource enjoys an exactly symmetrical universal comparative advantage in one demand and all demands and resource supplies are similarly symmetrical, then each resource will specialize in one and only one demand until simultaneous exhaustion.

In the general case, results are more limited, but the same tendencies prevail. Within demands, resources are used in strict order of absolute advantage. Universally dominant resources are initially employed in all demands, provided that they are sufficiently abundant. Strictly inferior resources are exclusively used in the last stage of resource use. In the polar case, an unlimited quantity of a strictly inferior resource is a backstop resource. With an equal number of resources and demands, if each resource has absolute advantage in a given demand, no resource could exclusively supply all demands. As this case converges towards perfect symmetry, the use profile converges to a single stage solution, wherein each resource is used only according to comparative advantage. On the other hand, where resources can be ranked according to dominance (one resource with universal dominance, the next with universal dominance over the rest), then resource use converges to least-cost-first as conversion costs across demands become decreasingly disparate.

With only two resources and two demands, either each resource has an absolute advantage in one end use or one resource is dominant. The possible sequences of resource use are characterized according to resource scarcity and the costs of extraction and conversion, thus highlighting the differentiated roles of absolute and comparative advantage as well as resource abundance. When a resource has a dominant absolute advantage and is sufficiently abundant, it is used exclusively at the beginning, contrary to the principle of comparative advantage. For example, oil may be used for electricity initially even though coal has comparative advantage in that demand. However, if each resource has absolute advantage in a given use, then resources are always used according to comparative advantage, and no resource may be an exclusive supplier, except in the terminal stage.

Taxes on a dirty resource or on a given sector may induce significant substitution effects, including substitution towards sectors not being taxed, or a complete switch in resource use between sectors. For example, a resource and sector-specific tax (e.g., a gasoline tax) may shift comparative advantage such that oil is used in electricity and coal in transportation. The implication is that reduction of oil consumption (e.g., to reduce dependence on imports) or of coal consumption (e.g., to reduce

carbon emissions) may entail taxing those resources in *all* uses. More generally, evaluating the long-run consequences of alternative energy tax proposals requires calculation of the full energy use trajectory for each of the alternatives.

The explanatory power of the neo-Ricardian theory described above can be enhanced by extending the model to include demand shifts, technological change, policy distortions, and transportation costs. For example the transition from whale oil to petroleum in the late nineteenth century has been attributed to the invention of the kerosene lantern. The transition from coal to oil in the late 19th and early 20th centuries is said to be due to advantages in transportation, the advent of the automobile, and technological improvements that lowered extraction and conversion costs.<sup>27</sup> Further research is also needed to characterize higher dimensional models more fully.

One can envision a similar theory being developed for explaining patterns of international trade. Dynamic comparative advantage has long been linked to changing endowments of factors. Inasmuch as resource depletion and capital accumulation are two sides of the same capital–theoretic coin, one expects the trade-off between dynamic and intersectoral specialization to carry over to neoclassical trade theory.

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## Appendix A

**Proof of Proposition 1.** We adopt Farzin’s (1982) proof of Theorem 15 in Seierstad and Sydsaeter (1987, Chapter 3) to show that a unique optimal solution exists to (1)–(3) and Theorem 13 to show that the necessary conditions are also sufficient. Define  $f_0(d_{ij}(t), t | i \in R, j \in U)$  as the integrand of (1) and  $f_i(d_{ij}(t) | j \in U) = -\sum_{j \in U} d_{ij}(t)$ ,  $i \in R$ . Then we need to prove the following statements:

1. *The functions  $f_0(d_{ij}(t), t | i \in R, j \in U)$  and  $f_i(d_{ij}(t) | j \in U)$ ,  $i \in R$  are continuous.*

**Proof.** By inspection.  $\square$

2. *The functions  $d_{ij}(t)$ ,  $i \in R, j \in U$  are bounded.*

**Proof.** Since  $D_j(p)$  is bounded by assumption for any given  $j$ ,  $d_{ij}(t)$  is bounded, each  $i \in R$ . This is true for all  $j \in U$ .  $\square$

<sup>27</sup>See e.g., Rhodes (2002). The extent to which these changes were induced may also be the subject of further investigation.

3. For any admissible path  $\{d_{ij}(t) \mid i \in R, j \in U\}$ , there exists a piecewise continuous function  $\phi_0(t)$  with  $\int_{t_0}^{\infty} \phi_0(t) dt < \infty$  such that  $f_0(d_{ij}(t), t \mid i \in R, j \in U) \leq \phi_0(t)$ .

**Proof.** Since  $\int_{t_0}^{\infty} D_j(p) dp < \infty, j \in U, \int_{t_0}^{\infty} \sum_{i \in R} d_{ij}(t) D_j(p) dp$  is bounded for all  $j \in U$ . Since all other terms in the integrand of (1) are bounded,  $e^{rt} f_0(d_{ij}(t), t \mid i \in R, j \in U) < K$ , where  $K$  is some upper bound. Let  $\phi_0 = Ke^{-rt}$  so that  $\int_{t_0}^{\infty} \phi_0(t) dt = K/r < \infty$ . Then  $f_0(d_{ij}(t), t \mid i \in R, j \in U) \leq Ke^{-rt} = \phi_0(t)$ .  $\square$

4. For any admissible  $\{d_{ij}(t) \mid i \in R, j \in U\}$ , there exists piecewise continuous functions  $\phi_i(t), i \in R$  with  $\int_{t_0}^{\infty} \phi_i(t) dt < \infty$  such that  $|f_i(d_{ij}(t) \mid j \in U)| \leq \phi_i(t), \forall i \in R$ .

**Proof.** Let  $\phi_i(t) = -\dot{Q}_i(t)$ . Then

$$\int_{t_0}^{\infty} \phi_i(t) dt = - \int_{t_0}^{\infty} \dot{Q}_i(t) dt = - \lim_{t \rightarrow \infty} Q_i(t) + Q_i(t_0) \leq Q_i(t_0) \text{ since } Q_i(t) \geq 0 \text{ from (2).}$$

Now  $d_{ij}(t) \geq 0$  implies from (3) that  $\dot{Q}_i(t) \leq 0$  hence  $|f_i(d_{ij}(t) \mid j \in U)| = |-\sum_{j \in U} d_{ij}(t)| = |\dot{Q}_i(t)| = -\dot{Q}_i(t) = \phi_i(t)$ . This holds for all  $i \in R$ .  $\square$

5. For any admissible  $\{d_{ij}(t) \mid i \in R, j \in U\}$ , there exist non-negative functions  $a(t)$  and  $b(t)$  such that  $\|f_1(d_{ij}(t)), f_2 d_{ij}(t), \dots\| \leq a(t) \|Q_1(t), Q_2(t), \dots\| + b(t)$ .

**Proof.** Since  $f_i(d_{ij}(t)) = -\sum_{j \in U} d_{ij}(t)$  by definition, it is bounded by statement 2,  $\forall i \in R$ . Thus the norm  $\|f_1, f_2, \dots\| = (f_1^2 + f_2^2 + \dots)^{1/2}$  is bounded. Let  $b(t)$  be this bound and put  $a(t) = 0$ .  $\square$

6. The function  $f_0(d_{ij}(t), t \mid i \in R, j \in U)$  is concave for all  $t$ .

$D_j(p) < 0 \forall j \in U$  and linearity of the rest of the terms in (1) imply concavity of  $f_0 \forall t$ .  $\square$

**Proof of Proposition 10.** Suppose that  $q_{ac}(t_0) = 0$  for some  $c \in U$ . Then since demand is positive, there exists  $b \in R$  such that  $q_{bc}(t_0) > 0$ . Then from (6),  $w_{bc} + \lambda_b(t_0) = p_c \leq w_{ac} + \lambda_a(t_0)$  which since  $w_{bc} - w_{ac} \geq s_0$  implies that  $\lambda_a(t_0) \geq s_0 + \lambda_b(t_0)$  which from (8) yields

$$\lambda_a(t) = \lambda_a(t_0)e^{rt} \geq (s_0 + \lambda_b(t_0))e^{rt} = s_0e^{rt} + \lambda_b(t). \tag{A.1}$$

Since resource  $a$  is universally dominant,  $s_j > 0$  for  $j \in U$ . Let  $\hat{t}_j = \log(s_j/s_0)/r$  for  $j \in U$  so that

$$s_0e^{rt} > s_j \text{ for } t > \hat{t}_j \text{ and } j \in U. \tag{A.2}$$

Then from (A.1), (A.2) and the definition of  $s_j, w_{aj} + \lambda_a(t) \geq w_{aj} + s_0e^{rt} + \lambda_b(t) > w_{aj} + s_j + \lambda_b(t) \geq w_{bj} + \lambda_b(t)$  for  $t > \hat{t}_j$  and  $j \in U$ . So from (6),

$$q_{aj}(t) = 0 \text{ for } t > \hat{t}_j, j \in U. \tag{A.3}$$

Let  $\gamma_j(t) = w_{aj} + s_0e^{rt}$  for  $j \in U$  so that

$$\gamma_j(t_0) = w_{aj} + s_0, \quad \gamma_j(\hat{t}_j) = w_{aj} + s_j, \quad \text{and} \quad \gamma_j'(t) = rs_0e^{rt} = r(\gamma_j(t) - w_{aj}) \text{ for } j \in U. \tag{A.4}$$

From (A.1) and since  $\lambda_b(t) \geq 0$ ,

$$w_{aj} + \lambda_a(t) \geq w_{aj} + s_0 e^{rt} + \lambda_b(t) \geq w_{aj} + s_0 e^{rt} = \gamma_j(t) \quad \text{for } j \in U. \tag{A.5}$$

From (6),

$$\begin{aligned} p_j(t) = w_{aj} + \lambda_a(t) &\Rightarrow D_j(w_{aj} + \lambda_a(t)) = D_j(p_j(t)) \geq q_{aj}(t) \quad \text{for } j \in U \quad \text{and} \\ p_j(t) < w_{aj} + \lambda_a(t) &\Rightarrow q_{aj}(t) = 0 \leq D_j(p_j(t)) \quad \text{for } j \in U, \quad \text{and therefore} \\ D_j(w_{aj} + \lambda_a(t)) &\geq q_{aj}(t) \quad \text{for } j \in U. \end{aligned} \tag{A.6}$$

Thus

$$\begin{aligned} \sum_{j \in U} \int_{s_0 + w_{aj}}^{s_j + w_{aj}} \frac{D_j(p)}{(p - w_{aj})} dp &= \sum_{j \in U} \int_{\gamma_j(t_0)}^{\gamma_j(\hat{t}_j)} \frac{D_j(p)}{(p - w_{aj})} dp = r \sum_{j \in U} \int_{t_0}^{\hat{t}_j} D_j(\gamma(t)) dt \\ &\geq \sum_{j \in U} \int_{t_0}^{\hat{t}_j} D_j(w_{aj} + \lambda_a(t)) dt \\ &\geq r \sum_{j \in U} \int_{t_0}^{\infty} q_{aj}(t) dt = r Q_a(t_0) \end{aligned} \tag{A.7}$$

where the change of variables follows from (A.4), the first inequality follows from (A.5) and that demand is downward sloping, the second inequality follows from (A.3) and (A.6), and the last equality follows from (4). Since (A.7) contradicts the premise, the supposition is false and so  $q_{ac}(t_0) > 0$ . Then since  $c$  was arbitrary,  $q_{aj}(t_0) > 0$  for  $j \in U$ .  $\square$

**Proof of Proposition 15.** Define  $\phi_E(t) = p_{OE}(t) - p_{CE}(t)$  and  $\phi_T(t) = p_{OT}(t) - p_{CT}(t)$ . Then  $\phi_E(t) - \phi_T(t) = (p_{OE} - p_{OT}) + (p_{CT} - p_{CE}) = (w_{CT} - w_{OT}) + (w_{OE} - w_{CE}) = k$ . Thus  $\phi_E(t) = (\lambda_O + w_{OE}) - (\lambda_C + w_{CE}) = (\lambda_O - \lambda_C) + (w_{OE} - w_{CE}) = (\lambda_O(t_0) - \lambda_C(t_0))e^{rt} + (w_{OE} - w_{CE})$ . Similarly,  $\phi_T(t) = (\lambda_O(t_0) - \lambda_C(t_0))e^{rt} + (w_{OT} - w_{CT})$ . Suppose  $\lambda_O(t_0) \leq \lambda_C(t_0)$ . Then  $\phi_E(t) < 0, \phi_T(t) < 0$  so that coal is never extracted, contradicting Lemma 1. Thus  $\lambda_O(t_0) > \lambda_C(t_0)$  and  $\Phi_E(t), \Phi_T(t)$  are continuous and  $\dot{\phi}_E(t) > 0, \dot{\phi}_T(t) > 0$ . If  $\phi_E(t_0) \geq k$ , then  $\phi_T(t_0) = \phi_E(t_0) - k \geq 0$  so that oil will never be used. Hence  $\phi_E(t_0) < k$ . Let  $t_0 = t_1$  where  $t_1$  denotes the switch point where electricity supply moves from oil to coal. Then,  $\phi_E(t_0) \geq 0$ . Since  $\Phi_E(t)$  is monotone increasing,  $\phi_E(t) = p_{OE}(t) - p_{CE}(t) > 0$  for all  $t \in (t_0, \infty)$  it implies that  $q_{OE}(t) \equiv 0$ . Thus  $\lambda_O(t_0) - \lambda_C(t_0) \geq w_{CE} - w_{OE}$  so that  $\phi_E(t) \geq (w_{CE} - w_{OE})(e^{rt} - 1)$ . Define

$$t_N = \left( \log \left( 1 + \frac{k}{w_{CE} - w_{OE}} \right) \right) / r.$$

Then for  $t \geq t_N$ ,  $\phi_E(t) > (w_{CE} - w_{OE})(e^{rt_N} - 1) = k$  so that  $\phi_T(t) = \phi_E - k > 0$ , hence  $q_{OT}(t) = 0$  for  $t \geq t_N$ . Let  $\gamma(t) = w_{OT} + (w_{CE} - w_{OE})e^{rt}$ . Then  $\gamma(t_0) = w_{CT} - k$ ,

$\gamma(t_N) = w_{CT}$  and  $\gamma'(t) = r(\gamma(t) - w_{OT})$ . Finally,

$$\int_{w_{CT-k}}^{w_{CT}} \frac{D_T(p)}{p - w_{OT}} dp = \int_{\gamma(t_0)}^{\gamma(t_N)} \frac{D_T(p)}{p - w_{OT}} dp = \int_{t_0}^{t_N} \frac{D_T(\gamma(t))}{\gamma(t) - w_{OT}} \gamma'(t) dt,$$

$$r \int_{t_0}^{t_N} D_T(\gamma(t)) dt \geq r \int_{t_0}^{t_N} D_T(p_{OT}(t)) dt \geq r \int_{t_0}^{\infty} d_{OT}(t) dt = rQ_0(t_0). \quad \square$$

### Appendix B

We show that Corollary 1 of Benveniste and Scheinkman (1979) holds for the problem defined in Eqs. (1)–(3). Our maximization problem in (1)–(3) can be rewritten in their notation as:

Given a technology set  $T \subseteq \mathfrak{R}^{2m}$ , find an absolutely continuous path  $(Q_i(t))$  which solves

$$\text{Max} \int_0^{\infty} u(Q_i(t), \dot{Q}_i(t), t) dt$$

subject to  $(Q_i(t), \dot{Q}_i(t), t) \in T \forall t$  and  $Q_i(t_0)$  fixed, where  $Q_i = (Q_1, \dots, Q_m)$  is the vector of resource stocks and  $\dot{Q}_i(t) = (\sum_{j=1}^n q_{1j}, \dots, \sum_{j=1}^n q_{mj})$ . We check that the following conditions hold:

**Assumption 1.** The pair  $(Q(t), \dot{Q}(t)) \in T \subseteq \mathfrak{R}_+^{2m}$  is convex since resource stocks are non-negative and finite and  $T$  is not empty since there must be a resource with a positive stock.

**Assumption 2.** The mapping  $u : T \times R \rightarrow R$  given by

$$u(\cdot, \cdot, t) = e^{-rt} \left[ \sum_{j \in U} \int_0^{\sum_{i \in R} q_{ij}(t)} D_j^{-1}(x) dx - \sum_{i \in R} \sum_{j \in U} w_{ij} q_{ij}(t) \right]$$

is continuously differentiable on  $\overset{0}{T} \times R$  since the demand function is continuous, and  $u(\cdot, \cdot, t) : T \rightarrow \mathfrak{R}$  is concave since the right-hand side of the above equation is concave (see Theorem 10.7, Rockafellar, 1970).

**Assumption 3.** An optimal solution  $\{y(t, Q_i(t_0))\}_{t=t_0}^{\infty}$  exists and  $V(Q_i)$  is well-defined in some neighborhood of  $Q_i(t_0)$  (see Proposition 1).

**Assumption 4.**  $(Q(t_0), \dot{y}(t_0, Q_i(t_0))) \in \overset{0}{T}$  and the optimal control is a piecewise function of time (see Seierstad and Sydsaeter, 1987, Theorem 1, p. 75 and Theorem 12, p. 234).

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