Discontinuous Extraction of a Nonrenewable Resource

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Abstract

This paper examines the sequence of optimal extraction of nonrenewable resources in the presence of multiple demands. We provide conditions under which extraction of a nonrenewable resource may be discontinuous over the course of its depletion.

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1. Introduction

A fundamental result in resource economics, the *Herfindahl rule*, is that when there is a single demand, extraction of identical deposits of a nonrenewable resource should be in the order of their unit costs of extraction (e.g., Herfindahl (1967), Solow and Wan (1976), Lewis (1982)). However, using a model of trash hauling between cities (demands) and landfills with a fixed capacity (resources), Gaudet, Moreaux and Salant (2001) prove a "vacillation" result: in the presence of setup costs a city may temporarily abandon a low marginal cost site, move to a higher cost site and then return to the former at a later date. An implication of this result is that a nonrenewable resource may be extracted discontinuously, i.e., over two disjoint time periods. In this paper, we show that discontinuous extraction of a nonrenewable resource is still possible, *even without setup costs*. We provide conditions for this discontinuity to occur.

We modify the framework of Chakravorty and Krulce (1994, henceforth CK) who consider two nonrenewable resources, oil (*O*) and coal (*C*) and two demands, electricity (*E*) and transportation (*T*). We add a third backstop resource (*B*) with an infinite supply (e.g., solar power).[2] While the assumption of a constant unit extraction cost in CK is retained for each resource (c_i, $i = O, C, B$) we specify conversion costs as both resource and demand-specific (z_{ij}, $i = O, C, B$; $j = E, T$) so that the net cost of resource i in demand j is $w_{ij} = c_i + z_{ij}$.

[2] At least three resources are needed for discontinuous extraction with two demands. Amigues *et al* (1998) provide a single-demand case wherein the Herfindahl rule is violated but with no discontinuous extraction of a resource.

The planner maximizes the discounted social surplus W with respect to $q_{ij}(t)$, extraction rates of resource i in demand j:

$$W = \int_0^\infty e^{-rt}\left[\sum_j\left(\int_0^{\sum_i q_{ij}} D_j^{-1}(x)dx\right) - \sum_{i,j}(c_i + z_{ij})q_{ij}(t) - \sum_i \lambda_i(t)\sum_j q_{ij}(t)\right]dt \tag{1}$$

subject to

$$q_{ij}(t) \geq 0; \quad Q_i(t) \geq 0; \quad \dot{Q}_i(t) = -\sum_j q_{ij}(t)$$

where r denotes the discount rate, D_j^{-1} the inverse demand function for j, $Q_i(t)$ the stock of resource i available at time t and $\lambda_i(t)$ the co-state variable for resource i. Define the equilibrium price for demand j as $p_j(t) = D_j^{-1}(\sum_i q_{ij}(t))$ and the price of resource i in demand j as $p_{ij}(t) = c_i + z_{ij} + \lambda_i(t) \equiv w_{ij} + \lambda_i(t)$. The necessary and sufficient conditions[3] are

$$p_j(t) \leq p_{ij}(t) \quad \text{(if < then } q_{ij}(t) = 0\text{)}\ ; \tag{2}$$

$$\dot{\lambda}_i(t) = r\lambda_i(t)\,; \tag{3}$$

$$\lim_{t\to\infty} e^{-rt}\lambda_i(t) \geq 0\,; \quad \lim_{t\to\infty} e^{-rt}\lambda_i(t)Q_i(t) = 0\,. \tag{4}$$

[3] The proof of sufficiency is essentially the same as in CK, hence suppressed.

Conditions (3) and (4) imply that $\lim_{t\to\infty} Q_i(t) = 0$ for nonrenewable resource i, $i = O, C$, and $\lambda_B(0) = \lambda_B(t) = 0$ for the backstop resource which is in infinite supply.

2. Optimal Extraction Sequence

Consider the case in which oil is the cheapest resource for both demands and the backstop is the most expensive. That is,

Assumption: $0 < w_{Oj} < w_{Cj} < w_{Bj} < \infty, \quad j = E, T$. (5)

As shown by Chakravorty, Krulce, and Roumasset (2005), in general the ordering of the shadow prices is exactly the reverse of that of net costs, hence $0 = \lambda_B(t) < \lambda_C(t) < \lambda_O(t) < \infty \quad \forall \; t \in [0, \infty)$. Since oil and coal are nonrenewable resources, they will be eventually exhausted and the backstop is used for both demands. By virtue of their Proposition 7, the extraction sequence in each demand follows the order of the net costs, i.e., oil followed by coal and then by the backstop resource (see Fig.1). Not every resource needs to be extracted for each demand.

3. Conditions for Discontinuous Resource Extraction

In this section, we demonstrate graphically the possibility of discontinuous extraction of a nonrenewable resource, and then provide necessary and sufficient conditions for the discontinuity to occur. In Fig.1, the (energy) resource price for each demand is depicted as an envelope curve: in bold solid for transportation and in bold dash for electricity. Coal is extracted in phase II and again in phase IV, but *not* in the intermediate phase III.

<**Figure 1 here**>

The switch point sequence in Fig. 1 is *S1*: $0 < t_{1E} < t_{2E} < t_{1T} < t_{2T} < \infty$ where t_{1j} and t_{2j} denote, respectively, oil-to-coal and coal-to-backstop switch points. If the $p_{OE}(t)$ curve in the lower part of Fig. 1 shifts up, oil may not be used in electricity, but coal extraction will remain discontinuous with an altered switch point sequence *S2*: $t_{1E} \le 0 < t_{2E} < t_{1T} < t_{2T} < \infty$. Either *S1* or *S2* is equivalent to the following three inequalities:

$$(i).\ 0 < t_{2E};\ (ii).\ t_{2E} < t_{1T};\ (iii).\ t_{1j} < t_{2j}, \quad j = E, T. \tag{6}$$

Under these two sequences, coal is extracted first for electricity and then for transportation. We can now state

PROPOSITION: *Given the Assumption in* (5)*, coal is extracted discontinuously, first for electricity* (*E*) *and then for transportation* (*T*) *after a time delay, iff*

(I). $\lambda_C(0) < w_{BE} - w_{CE}$;

(II). $1+\max_j\left\{\frac{w_{Cj}-w_{Oj}}{w_{Bj}-w_{Cj}}\right\} < \frac{\lambda_O(0)}{\lambda_C(0)} < 1+\frac{w_{CT}-w_{OT}}{w_{BE}-w_{CE}}$, $j=E,T$.[4]

Proof: At the switch points for demand *j*, $p_{Oj}=p_{Cj}$ and $p_{Cj}=p_{Bj}$ which yield:

$$t_{1j}=\frac{1}{r}\ln\frac{w_{Cj}-w_{Oj}}{\lambda_O(0)-\lambda_C(0)};\quad t_{2j}=\frac{1}{r}\ln\frac{w_{Bj}-w_{Cj}}{\lambda_C(0)}\ . \tag{7}$$

Substituting (7) into (6), noting $0<\lambda_C(0)<\lambda_O(0)<\infty$ from Section 2, we can rewrite (i), (ii) and (iii) in terms of $\lambda_i(0)$ and w_{ij}:

$$(i).\ 0<t_{2E} \quad\Leftrightarrow\quad \lambda_C(0)<w_{BE}-w_{CE}\ ; \tag{8}$$

$$(ii).\ t_{2E}<t_{1T} \quad\Leftrightarrow\quad \frac{\lambda_O(0)}{\lambda_C(0)}<1+\frac{w_{CT}-w_{OT}}{w_{BE}-w_{CE}}; \tag{9}$$

$$(iii).\ t_{1j}<t_{2j} \quad\Leftrightarrow\quad \frac{w_{Cj}-w_{Oj}}{\lambda_O(0)-\lambda_C(0)}<\frac{w_{Bj}-w_{Cj}}{\lambda_C(0)} \quad\Leftrightarrow$$

$$\frac{\lambda_O(0)}{\lambda_C(0)}>1+\max_j\left\{\frac{w_{Cj}-w_{Oj}}{w_{Bj}-w_{Cj}}\right\}. \tag{10}$$

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[4] There exists a subset of $w=(w_{OE,}w_{CE},w_{BE},w_{OT,}w_{CT},w_{BT})$ which admits conditions (I) and (II), e.g., $w=(4,5,6,1,4,6)$. Given w, $\lambda_O(0)$ and $\lambda_C(0)$ still depend on other factors such as the initial stocks of resources, the discount rate and the magnitude of demands, hence are not determined solely by w.

Noting that (9) and (10) jointly are equivalent to (II) in the Proposition completes the proof. ■

The conditions in the Proposition can be re-expressed as three simple inequality constraints: $\lambda_C(0) < \alpha$, $\lambda_O(0) < \beta\lambda_C(0)$ and $\lambda_O(0) > \gamma\,\lambda_C(0)$ where

$$\alpha \equiv w_{BE} - w_{CE} > 0 \;; \tag{11}$$

$$\beta \equiv 1 + \frac{w_{CT} - w_{OT}}{w_{BE} - w_{CE}} > 1; \tag{12}$$

$$\gamma \equiv 1 + \max_j \left\{ \frac{w_{Cj} - w_{Oj}}{w_{Bj} - w_{Cj}} \right\} > 1, \tag{13}$$

which are graphically depicted in Fig. 2. Let $\mathrm{F} = \{(\lambda_C(0), \lambda_O(0)) \,|\, 0 < \lambda_C(0) < \lambda_O(0) < \infty\}$ which lies above the 45° line in the figure. Set F defines the domain for $(\lambda_C(0), \lambda_O(0))$ on which the discontinuous extraction of coal is *feasible*. The entire shaded area (S) represents an open subset that satisfies the conditions for either *S1* or *S2* to occur. Note that $t_{1E} > 0$ for *S1* and $t_{1E} \le 0$ for *S2*. Using t_{1j} in (7) for $j = E$, we can restate these two inequalities as $\lambda_O(0) > \mu + \lambda_C(0)$ for *S1* and $\lambda_O(0) \le \mu + \lambda_C(0)$ for *S2*, respectively, where $\mu \equiv w_{CE} - w_{OE} > 0$. The line $\lambda_O(0) = \mu + \lambda_C(0)$ splits the entire shaded area in the figure into two parts. The dark shaded area (S_1) is an open subset of S that admits only sequence *S1* and the light shaded area ($\mathrm{S} - \mathrm{S}_1$) admits only *S2*. If $\mu \ge \mu^* \equiv \alpha(\beta - 1)$, there exists no $\left(\lambda_C(0), \lambda_O(0)\right) \in \mathrm{S}$ which admits *S1*, so that the entire shaded area admits only *S2*.

<Figure 2 here>

The Proposition is symmetric with respect to *E* and *T*. Fig.1 shows that if *E* and *T* were interchanged, coal would still be extracted discontinuously with *S1* and *S2* redefined as $0 < t_{1T} < t_{2T} < t_{1E} < t_{2E} < \infty$ and $t_{1T} \leq 0 < t_{2T} < t_{1E} < t_{2E} < \infty$, respectively.

4. Conclusion

This paper provides conditions under which optimal extraction of a nonrenewable resource is discontinuous. At least three resources and two demands are necessary for discontinuous extraction to occur. With many resources and demands, extraction patterns that appear chaotic may be consistent with efficient resource use.

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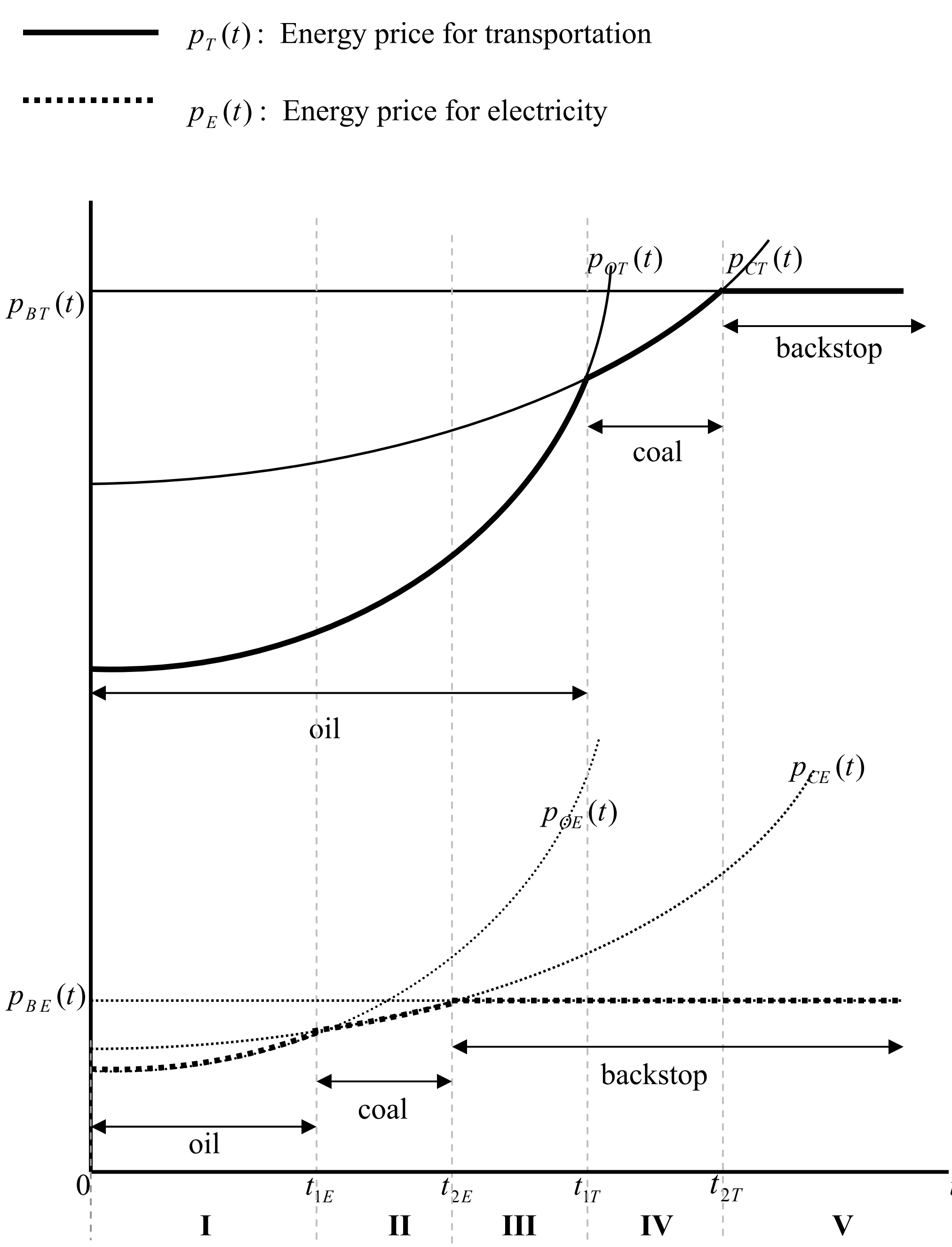

Fig. 1: Discontinuous Coal Extraction: Coal is extracted in phase II and IV, but not in III.

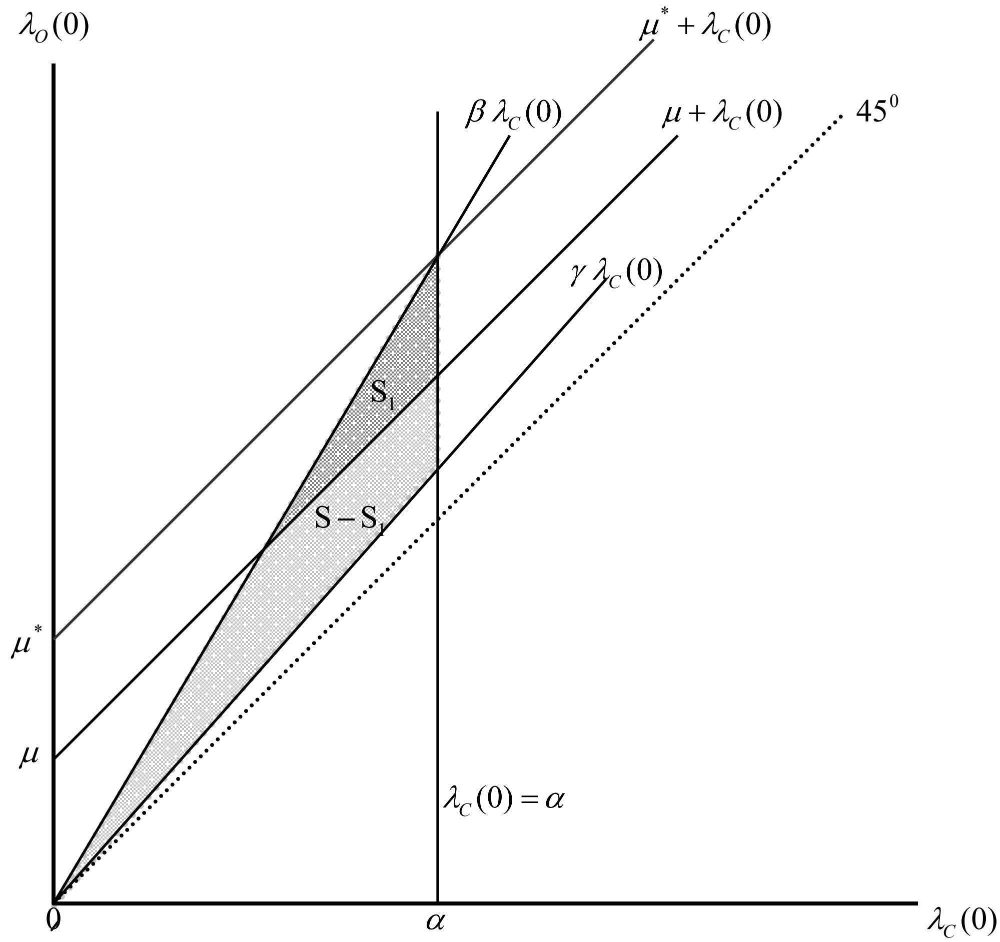

Fig. 2: Coal Extraction is Discontinuous on the Shaded Open Set