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Precision in Accounting Information, Financial Leverage and the Value of Equity

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Abstract: Using an equity valuation model characterized by periodic imperfect accounting information, we examine how financial leverage affects a firm's accounting quality choice (i.e., precision). We find that the existence of financial leverage motivates firms with average to good performance to prepare accounting information with a high degree of precision. However, we conclude that when a firm is performing poorly it has an incentive to reduce accounting precision in order to lower the likelihood of both a debt covenant violation and the detection of accounting bias.

Keywords: leverage, accounting precision, value of equity

1. INTRODUCTION

We examine the effect of financial leverage on shareholders' incentives to provide greater precision in accounting information. We are concerned with the fundamental effect that differing levels of debt and equity have on the quality of accounting information (i.e., precision).[1] While our analysis is performed from the perspective of equity holders, the rational anticipation and reaction of debt holders to the equity holders' accounting quality decision can be easily disentangled. As a result, debt holders can properly price risky debt with imperfect accounting information. We begin from the work of Black and Scholes (1973), which demonstrated that equity may be valued as a contingent claim on the total value of the

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1 This paper is not solely about *bias* in accounting information *per se*.

firm.[2] In common with their paper, equity holders have a residual claim on the profit of the firm (residual to the claims of debt holders) and at maturity, debt becomes repayable. However, companies are not compelled to repay the debt. Rather, if the firm value is less than the debt obligation, the company can default with the lender effectively acquiring the company. In other words, equity holders have an abandonment option. We develop one-period and two-period contingent claims models where imperfect accounting information is used to value equity and/or to determine debt covenant violation.

Where a firm's value is measured by imperfect accounting information, equity holders can make two mistakes. First, equity holders may abandon a 'good' firm to debt holders (i.e., commit a type I error where a 'good' firm is defined as one with a 'true' value greater than the face value of its debt). This mistake will occur when the expected firm value is lower than the debt value, while the true firm value is higher than the debt value. Second, equity holders may retain a 'bad' firm (i.e., commit a type II error where a 'bad' firm is defined as one with a 'true' value less than the face value of its debt). This mistake will occur when the expected firm value is higher than the debt value, while the true firm value is lower than the face value of debt. Both types of mistakes, abandoning a good firm or retaining a bad firm, shift value from equity holders to debt holders. Increasing the precision of accounting information will reduce the likelihood of both types of mistakes at debt maturity, thus increasing the value of equity. As a result, at debt maturity increasing precision unambiguously increases the welfare of equity holders. Debt holders must be able to anticipate this in order to properly price the debt at the time of loan origination.

Equity holders may be required to make the abandonment decision (i.e., repay or abandon) before debt maturity if a debt covenant has been breached in an intermediate period.[3] The Black and Scholes (1973) equity valuation model was extended by Black and Cox (1976) and Leland (1994) to include debt covenants. These papers assumed, as had Black and Scholes (1973), that firm value could be perfectly observed. Relaxing this assumption, the covenants in our model are measured by noisy accounting information. Specifically, where a firm's accounting value is less than the covenant value, debt holders may impose a cost to equity holders, such as an increased interest rate.[4] Therefore, in intermediate periods with debt covenants, accounting information may trigger a violation of the covenant which is costly to equity holders. Precision in accounting information therefore affects both whether a covenant is breached and the later abandonment decision. We will demonstrate that the expected value of equity increases in accounting precision and that improving accounting precision will have the greatest impact on equity valuation when financial leverage is high (i.e., where the face value of debt is high relative to the total firm value), and when the firm's value is close to the debt covenant value (i.e., in proximity to breaching a covenant). When a firm is

2 Equity may be valued as a call option where equity holders can purchase the firm at an exercise price equal to the face value of the debt, when the debt matures. A call option has limited liability in that its payoff is the difference between the realization of the underlying asset and the exercise price, but is never below zero. A put option insures the underlying asset from falling below the exercise price in that its payoff is the shortfall of the realization of the underlying asset from the exercise price.

3 With limited liability, equity holders have incentives to increase their option value at the expense of debt holders by increasing firm volatility (this follows directly from Black and Scholes, 1973). Core and Shrand (1999) argue that debt holders impose covenants in order to mitigate this problem.

4 Our results hold if debt holders demand immediate repayment after a debt covenant breach.

highly levered, the probability of a mistake (abandon a good firm or retain a bad firm) increases. A similar argument may be made for the relationship between firm value and the proximity to a debt covenant breach. When firm value is very high relative to the debt covenant threshold, precision has little effect on value (i.e., the firm's value is clearly above the debt covenant value). Similarly, when the firm value is very low relative to the debt covenant, precision also has little effect on the value of equity. However, for accounting values of the firm close to the debt covenant level, increasing accounting precision can significantly increase the value of equity.

If equity holders have private information about firm value, then high accounting precision is most desirable if the current value of the firm is moderately higher than the debt covenant level. That is, to significantly reduce the probability of a type two error, high precision is needed. Similarly, low accounting precision is most desirable if the current value of the firm is lower than the covenant level (because it is valuable to the equity holder that the probability of a type one error be high with respect to the determination of a debt covenant violation).

There are some basic assumptions underlying the models in this paper. Most important are those related to the distribution of the (unobserved) 'true' firm value and the (observed) accounting firm value. First, we make the common assumption used throughout the contingent claims literature that the true firm value will follow a random walk. Second, we assume that the accounting estimate of firm value is measured with noise. In examining the effects of accounting information on the value of equity, two dimensions are important: bias and precision (i.e., accuracy). The accounting literature has studied bias for two main reasons. First, accounting information is by construction biased. For example, the accounting principle of conservatism provides for an inherently downward estimate of value (e.g., see Antle and Nalebuff, 1991; Bradshaw and Sloan, 2002; and Watts, 2003). Second, the practice of accounting has a large degree of subjectivity. For example, accrual expectations, which are fundamental to our financial accounting system, are inherently subjective. This subjectivity can lead to the 'managing' of accounting information (i.e., earnings management), thus affecting equity value (e.g., see Arya, Glover and Sunder, 1998; Burgstahler and Dichev, 1997; Hribar and Collins, 2002; and Xie, 2001). Note that it is not just academics, but also regulators who have focused on the bias in accounting information. Following the collapse of Enron and WorldCom (attributed to irregularities and fraud), the Public Accounting Oversight Board was created and the Sarbanes-Oxley Act was enacted largely to reduce earnings manipulation (i.e., actions by management that introduce bias).[5]

Research on the effect of precision in accounting information on equity valuation is also well-established in the literature (for example, see Kim and Verrecchia, 1991a and 1991b; and Abarbanell et al., 1995). As in this paper, there is an assumption throughout the precision literature that accounting information is either unbiased, or that a transformation can be performed on the information to make it unbiased. While this abstracts from reality, it is a reasonable assumption given the focus of this paper. We examine a foundational relationship–the effect of accounting precision on

5 While there is no incentive to manage accounting numbers at the terminal date, there may be at the intermediate covenant date since it is only when a covenant is breached that equity holders must make an abandonment decision. Inflating accounting-based firm value will therefore increase equity value.

equity value through the fundamental claims of equity holders and debt holders to the firm (i.e., the residual claim by equity holders).[6]

There are significant implications from this study. When a leveraged firm has average to good performance, its accounting system is more likely to produce accurate and unbiased information. However, when a firm is performing poorly, it has an incentive to produce noisy and biased accounting information.

The remainder of the paper is organized as follows. Section 2 presents the basic assumptions underlying the model, describes our model, and analyzes the relationship between precision in accounting information and equity value when equity holders have no private information about firm value. Section 3 considers the impact of accounting information precision on the future value of the firm if equity holders have private information. We conclude in Section 4.

2. A MODEL WHERE EQUITY HOLDERS DO NOT POSSESS PRIVATE INFORMATION

We assume that all players are risk neutral. We assume that a firm's value at period t, A_t, follows a diffusion process represented by a geometric Brownian motion (a random walk with drift).[7] That is, $\ln \frac{A_t}{A_{t-1}}$ is normally distributed with a mean of μ and a variance of σ^2, where $\mu = r - \sigma^2/2$ and r is the risk-free interest rate. This assumption ensures that the value of the firm, A_t, cannot be negative (i.e., limited liability exists; see Cox and Rubinstein, 1985).

We assume that a firm's value cannot be perfectly measured, but rather there is an accounting system which can imperfectly measure the firm's value at the end of a period. The accounting value is initially assumed to be unbiased and later this assumption is relaxed by allowing bias to be incorporated into it.

Following from the above assumptions, the relationship between the accounting measure of firm value at time t, denoted as S_t, and the firm's 'true' value, A_t, is specified as follows:

$$S_t = A_t \bullet e^{x_t}, \tag{1}$$

where e^{x_t} is a noise term and x_t is normally distributed with a mean of $-\alpha_t^2/2$ and a variance of σ_t^2 (i.e., $x_t \sim N(-\frac{\alpha_t^2}{2}, \alpha_t^2)$). One property of equation (1) is that $E(S_t) = A_t$. In other words, accounting information at time t provides an unbiased estimate of the true firm value at time t. The level of precision in accounting information is equal to $1/\alpha_t$. To simplify the analysis we assume that, with certainty, the observed firm value equals the true firm value at time zero (i.e., $S_0 = A_0$). The face value of the firm's debt

6 Note that while Dhaliwal and Reynolds (1994) and Fischer and Verrecchia (1997) offer an *ex post* perspective on the relation between disclosure and limited liability, the purpose of our paper is to focus on an *ex ante* perspective: that is, a perspective that gives primary emphasis to the relation between precision and *anticipated* announcements.

7 For a brief discussion on the appropriateness of this assumption, see Black and Cox (1976).

at maturity is denoted as F. For convenience we assume that there exists only a single class of debt[8] and that priority of claims is not violated.[9]

To simplify the presentation, we introduce two additional variables, z_t and y_t, as follows:

$$z_t = \ln \frac{S_t}{A_0} \quad \text{and} \quad y_t = \ln \frac{A_t}{A_0}. \tag{2}$$

It follows from equation (1) that $z_t = y_t + x_t$.

(i) A One-Period Model

In our one-period model, debt matures at the end of the period. At this point, equity holders must make a decision whether to retain the firm and repay the debt holders (i.e., exercise an option to repay), or give the firm to the debt holders (i.e., allow the option to repay to expire).[10] Equity holders do not have perfect information regarding the firm's value and must make their decision based upon the accounting value of the firm.[11] Equity holders could liquidate the firm in order to avoid a type I error (that is, leave a good firm to debt holders). However, liquidation does not increase the equity holders' payoff for two reasons. First, outsiders will not pay a purchase price greater than the equity holder's expected value of the firm, since the outsiders' information is no more fine than the equity holders' information. With coarser information, the outsider's expected value of the firm is lower than the equity holders' expected value of the firm (this result is stated in Proposition 1 below). Second, deadweight liquidation costs may exist. Therefore, liquidation of the firm is not an efficient option and equity holders are better off either paying back the debt or leaving the firm to debt holders.[12]

Lemma 1: In a one period model, equity holders should repay the loan at maturity if the accounting value of the firm is greater than $S_1{}^*$, where $S_1{}^*$ has the following value:

$$S_1^* = A_0^{-\frac{\alpha_1^2}{\sigma^2}} \bullet F^{1+\frac{\alpha_1^2}{\sigma^2}} \bullet e^{-r\frac{\alpha_1^2}{\sigma^2}-\frac{\alpha_1^2}{2}}. \tag{3}$$

8 The purpose of the model in this paper is not to explain how firms set their level of borrowing but rather to consider the accounting precision choice for a given level of debt. Accordingly, we do not model the firm's choice of financial leverage, but rather we take it as given in order to make our model more tractable.

9 This assumption is generally implicit in modeling of financial instruments using a contingent claims framework. However, Weiss (1990) suggests that priority is not absolute. He finds that there exists some breakdown of priority of claims between unsecured creditors and equity holders, although it appears that priority does substantially hold.

10 Equity holders can increase their payoff by issuing dividends and increasing the variance of the firm's operations. However, allowing accounting based dividends is unlikely to affect our analysis and main conclusions. As a result, in order to streamline our analysis we assume that the firm is restricted by the debt contract from issuing any dividends or increasing the risk of the firm's operations. This assumption is consistent with the findings of Sweeny (1994) that firms often violate affirmative covenants but not negative covenants.

11 To avoid the possibility that equity holders and debt holders have different opinions about firm value, we assume that debt holders can conjecture the precision of the firm's accounting system and any possible bias. Alternatively, debt holders could rely on an external audit to confirm the firm's value.

12 A third alternative would be for the firm to file for bankruptcy protection in order to delay the debt holder's right to liquidate the firm. For simplicity, we assume that this option has already been exhausted by the firm prior to period one.

Otherwise, equity holders should leave the firm to the debt holders. We label $S_1{}^*$ as the exercise accounting value.

Proof: See the Appendix.□

Equation (3) presents a threshold accounting value at which the equity holders are indifferent between retaining the firm or leaving the firm to the debt holders at the end-of-period debt maturity date. S_1^* is the observed accounting value where the expected firm value at the end of the period equals the face value of the debt. From (3), it is evident that S_1^* is an increasing function of the face value of the debt F and a decreasing function of the firm's initial value A_0. In other words, the degree of financial leverage will affect the exercise accounting value S_1^*. The intuition for S_1^* being an increasing function of F is as follows. When all other factors are given, an increase in debt requires a higher cut-off expected intrinsic value, P, for the firm to pay back the loan, which in turn requires a higher cut-off accounting value, S_1^*, since P is an increasing function of S_1. The intuition for S_1^* being a decreasing function of A_0 is as follows. When the accounting value is a noisy measure of the firm's intrinsic value, the calculation of the expected intrinsic value of the firm is based on both the accounting value and the prior distribution of the intrinsic value. A decrease in A_0 directly lowers the prior expected intrinsic value. To obtain the same expected intrinsic value (that equals F, a constant), a higher accounting value is required to compensate for the reduced contribution of a lower prior intrinsic value to the calculation of the expected firm value. Alternatively, an increase in A_0 results in a lower accounting value being required to achieve the same expected intrinsic value.

From (3) we have:

$$\frac{\partial S_1^*}{\partial \alpha_1} = 2\alpha_1 S_1^* \left(\frac{1}{\sigma^2} \ln \frac{F}{A_0 e^r} - \frac{1}{2} \right) < 0. \tag{4}$$

Inequality (4) holds since the face value of the debt should be lower than the expected firm value. That is, $F < A_0\, e^{r+\frac{\sigma^2}{2}}$ and the exercise accounting value is an increasing function of the precision in accounting information. The intuition for S_1^* being a decreasing function of α_1 is as follows. When all other factors are given, increasing the accounting noise (α_1) reduces the impact of the accounting value in the calculation of the expected intrinsic firm value. More specifically, with greater noise the accounting value on either side of the mean is discounted more, such that the expected value is moved toward the centre. Since F is lower than the prior expected firm value at period end, the cut-off accounting value is lower than the mean of the distribution. Consequently, for a given F, the cut-off accounting value is lower with a higher α_1.

Lastly, S_1^* is a decreasing function of σ^2. Since F is lower than the prior expected firm value at period end, when all other factors are given, increasing the operational uncertainty (σ^2) reduces the impact of the prior expectation of the intrinsic value on the calculation of the expected intrinsic value. Therefore, increasing σ^2 requires a higher cut-off accounting value to compensate for the reduced impact of the prior distribution of the value, in order to obtain the same expected firm value (which equals F). Note also that when σ^2 declines, the prior distribution plays a more important role in the calculation of expected intrinsic value since the debt amount (F) is lower than the expected firm value ($A_0 e^{\gamma}$). Therefore, when the accounting value plays a reduced role in the calculation of expected intrinsic value, the benefit associated with its precision is reduced accordingly.

Lemma 2: When the firm's true economic value is not observable and equity holders must rely on noisy accounting information, then at the beginning of the period the expected value of equity can be expressed as follows:

$$V_1 = A_0 \Phi \left[\frac{\frac{\alpha_1^2}{\sigma^2} \ln \frac{A_0 e^r}{F} + \ln \frac{A_0}{F} + \mu + \sigma^2}{\sqrt{\alpha_1^2 + \sigma^2}} \right] - F e^{-r} \Phi \left[\frac{\frac{\alpha_1^2}{\sigma^2} \ln \frac{A_0 e^r}{F} + \ln \frac{A_0}{F} + \mu}{\sqrt{\alpha_1^2 + \sigma^2}} \right] \quad (5)$$

where Φ is the operator of the cumulative density function of a standard normal distribution.

Proof: See the Appendix.□

The structure of equation (5) is similar to the Black-Scholes formula. However, imperfect accounting information introduces additional components into equation (5). The noise in accounting information (i.e., α_1) plays a role in determining the expected equity value. Proposition 1 presents the relationship between the expected value to equity holders and the precision in accounting information.

Proposition 1: For a given face value of debt, the expected equity value is an increasing function of the firm's precision in accounting information if information production is costless.

Proof: See the Appendix.□

This result is particularly interesting. Since the expected equity value is an increasing function of the variance of the firm's operations (Cox and Rubinstein, 1985), it would be reasonable to believe that the expected equity value would also be an increasing function of the variance of accounting information. However, this is not the case. An increase in the variance of a firm's operations means that equity holders receive the gain from possible positive movements in the firm's value and debt holders bear the loss from possible negative movements. Increasing the variance of accounting information does not increase or decrease the firm's value but increases the probabilities that the equity holders make a type I error (i.e., leave a good firm to debt holders) or type II error (i.e., keep a bad firm). Therefore, to increase the expected equity value, equity holders should increase the precision of accounting information.[13] If the cost of increasing precision in accounting information is trivial, then equity holders will set $\alpha_1 = 0$. As a result, the equity value represented by equation (5) becomes the same as the value determined by the Black-Scholes formula. However, if the cost of increasing precision is not trivial, then equity holders will set precision at an optimal level where equity value net of the cost of maintaining a certain level of precision is

13 Though a precision choice for accounting information is usually made by a firm's managers, equity holders can influence management's decision through their representatives on the board of directors and audit committee. For simplicity, we assume that equity holders make the precision decision directly. This assumption is consistent with Titman and Truman (1986) and Penno (1996) where equity holders and the management of the firm are assumed to have the same motivation to maximize equity value.

maximized.[14] In this paper, we assume that the cost of information production is an increasing function of accounting precision. The cost is denoted as $W(\alpha)$ and $W'(\alpha) < 0$. We analyze the marginal effect of increased precision on equity value to determine the optimal level of accounting precision in a firm. Note that we assume that equity holders can observe accounting precision, although this assumption is not essential for our results. For example, we could assume that the manager is the only person who determines and observes accounting precision. If we assume also that the manager's compensation is directly linked to the value of the firm at the end of the period, to maximize the end-of-period firm value the manager will maximize the firm value $V_1(\alpha) - W(\alpha)$ by choosing an appropriate α, which can be consistently anticipated by equity holders (investors). Therefore, though equity holders and potential investors do not directly observe accounting precision, they can anticipate it and the manager will choose the optimal precision on behalf of the equity holders.

Corollary 1: With a costly accounting system, accounting precision is an increasing function of debt.

Proof: See the Appendix.□

For a given level of precision, the expected costs to equity holders from possible type I and type II errors are increased when the face value of debt is increased (since the probability is higher that the observed accounting value is closer to the threshold accounting value). Therefore, the marginal benefit of improved accounting precision is an increasing function of the face value of debt. In our setting, the firm's initial value is fixed at A_0. Therefore, corollary 1 implies that accounting precision is an increasing function of the degree of financial leverage.

(ii) A Two-Period Model With No Interim Review of Debt Covenants

In this section we extend our one-period model to two periods. This allows us to better reflect the multi-period nature of debt. The debt of the firm matures at the end of period two and the firm calculates the accounting value at the end of each period. In this two-period model the debt holders do *not* perform an interim review at the end of period one (the accounting information at the end of period one is not used directly). The accounting information at the end of period two is used by equity holders to decide whether to keep the firm or leave the firm to the debt holders.

Lemma 3: Equity holders will repay debt at the end of period two if the accounting value of the firm at the end of period two is greater than S_2^*, where S_2^* has the following value:

$$S_2^* = A_0^{-\frac{\alpha_2^2}{\gamma^2}} F^{1+\frac{\alpha_2^2}{\gamma^2}} e^{-m\frac{\alpha_2^2}{\gamma^2}-\alpha_2^2} \qquad (6)$$

14 We assume that the cost of maintaining an accounting system is an increasing function of accounting precision, and analyze the marginal effect of increased precision on equity valuation in order to determine the optimal level of accounting precision in a typical firm. Since the cost of increasing precision is separate from production costs, to make the analysis tractable, we do not explicitly include the cost of maintaining an accounting information system in the model.

where:

$$m = \mu + \frac{\alpha_1^2\mu + \sigma^2 z_1 + \sigma^2\frac{\alpha_1^2}{2}}{\sigma^2 + \alpha_1^2} \tag{7}$$

$$\gamma^2 = \frac{\sigma^2(\sigma^2 + 2\alpha_1^2)}{\sigma^2 + \alpha_1^2}. \tag{8}$$

Otherwise, equity holders will abandon the firm to debt holders.

Proof: See the Appendix.□

The relationship between S_2^* and α_2 is similar to the relationship between S_1^* and α_1 discussed following Lemma 1. The equity value of the firm is determined by the following lemma.

Lemma 4: When the firm's true value is not observable and equity holders make their decision based on noisy accounting information, the expected value of equity can be expressed as follows:

$$V_2 = e^{-2r}\left(A_0\, e^{2\mu+\sigma^2}\Phi[d_2] - F\,\Phi[d_2 - C]\right) \tag{9}$$

where:

$$d_2 = \frac{A + \frac{\gamma^2}{\sqrt{\gamma^2+\alpha_2^2}} + B\left(\mu - \frac{\alpha_1^2}{2} + \sigma^2\right)}{\sqrt{B^2\left(\sigma^2 + \alpha_1^2\right) + 1}} \tag{10}$$

$$A = \frac{\frac{\gamma^2+\alpha_2^2}{\gamma^2\left(\sigma^2+\alpha_1^2\right)}\left(\mu\left(\sigma^2 + 2\alpha_1^2\right) + \sigma^2\frac{\alpha_1^2}{2}\right) + \frac{\alpha_2^2}{2} - \frac{\gamma^2+\alpha_2^2}{\gamma^2}\ln\frac{F}{A_0}}{\sqrt{\gamma^2 + \alpha_2^2}} \tag{11}$$

$$B = \frac{\sigma^2\left(\gamma^2 + \alpha_2^2\right)}{\gamma^2\left(\sigma^2 + \alpha_1^2\right)\sqrt{\gamma^2 + \alpha_2^2}} \tag{12}$$

$$C = \frac{\frac{\gamma^2}{\sqrt{\gamma^2+\alpha_2^2}} + B\sigma^2}{\sqrt{B^2\left(\sigma^2 + \alpha_1^2\right) + 1}} \tag{13}$$

and Φ is the operator of the cumulative density function of a standard normal distribution.

Proof: See the Appendix.□

The structure of equation (9) is similar to the Black-Scholes formula. However, in our model the imperfect accounting information at the end of both periods affects the expected equity value. If accounting information is perfect at the end of both periods, the value represented by (9) is the same as the value determined by the Black-Scholes formula in a two-period model. Proposition 2 presents the relationship between the expected equity value and the precision of accounting information at the end of periods one and two.

Proposition 2: For a given face value of debt, the expected equity value of a firm is an increasing function of the precision of accounting information for both periods one and two.

Proof: See the Appendix.□

When accounting precision increases, the expected equity value increases. Increasing precision in both periods reduces the uncertainty faced by equity holders at the end of period two. The discussion following Proposition 1 can also be applied to Proposition 2.

Corollary 2: It is more cost efficient to increase the precision of accounting information for period two than for period one.

Proof: See the Appendix.□

As there is no interim review, only the precision in accounting *value* at the end of period two will affect the abandonment decision. The precision in accounting information during period two directly affects the precision in accounting value at the end of period two, while the precision in accounting information during period one only indirectly affects the precision in accounting value at the end of period two (through its effect on accounting value at the end of period one). Therefore, to increase the precision in accounting value at the end of period two, it is more efficient and effective to increase the precision in accounting information for period two rather than for period one.

Corollary 3: If the precision in accounting information for period two is determined after observing the accounting value at the end of period one, then accounting precision is an increasing function of the accounting value at the end of period one (z_1) for $z_1 < z_1^*$ and a decreasing function of z_1 for $z_1 > z_1^*$, where:

$$z_1^* = \left(\frac{\sigma^2 + \alpha_1^2}{\sigma^2} \left(\ln \frac{F}{A_0} - \frac{\gamma^2 \alpha_2^{*2}}{2\left(\gamma^2 + \alpha_2^{*2}\right)} \right) - \frac{\left(\sigma^2 + 2\alpha_1^2\right)\mu}{\sigma^2} - \frac{\alpha_1^2}{2} \right) \tag{14}$$

and α_*^2 is the optimal accounting precision in the second period after observing z_1^*. The greatest accounting precision in the second period is reached when $z_1 = z_1^*$.

Proof: See the Appendix.□

Note that z_1^* is a function of F, the face value of debt. When the accounting value at the end of period one is high or low relative to the debt, the probability that equity holders make a type I or a type II error is low. First, if the accounting value at the end of period one is much higher than the face value of debt, the conditional probability that the actual firm value at the end of period two will be above the face value if debt is high. Therefore, the probability that equity holders make a type I error is low since the probability that equity holders would leave the firm to debt holders is low, and the probability that equity holders make a type II error is low since the probability that the firm is a 'bad' firm is low. Therefore, the benefit of having greater precision in the accounting report in the second period is not great since type I and type II error rates are already low. Second, if the accounting value at the end of period one is much lower than the face value of debt (i.e., very poor performance), the conditional probability that firm value at the end of period two is lower than the face value of debt, is high. Therefore, the probability that equity holders make a type I error is low since the probability that the firm is a 'good' firm is low, and the probability that the equity holders make a type II error is low since the probability that equity holders would pay back the debt is low.

If the accounting value at the end of period one is close to z^*, then the observed accounting value at the end of period two is more likely to be near the threshold accounting value. As a result, the probability that equity holders make a type I error or a type II error is high. The marginal benefit from increasing precision in accounting is maximized when the observed accounting value at the end of period one is z_1^*. Therefore, it is expected that precision in accounting information for period two is highest if the observed accounting value at the end of period one is z_1^*. If the observed accounting value at the end of period one is different from z_1^*, precision is lower and is a decreasing function of the distance between the observed accounting value and z_1^* in either direction.

The above result differs from both Titman and Truman (1986) and Penno (1996). Titman and Truman (1986) conclude that firms which privately anticipate a higher market value release more precise information whereas Penno (1996) shows that firms with poor prospects report more precise information. In both papers the accounting reports are signals to the market and the market conjectures the value of the firm based on the accounting report. In our setting, the firm's expected value of equity is affected by the precision in accounting information.

(iii) A Two-Period Model with an Interim Review of Debt Covenants

In this model, debt holders review the firm's accounting information at the end of period one.[15] If the firm's accounting value reported at the end of period one, S_1, is lower than a preset value, H, then the debt covenant has been breached.[16] The violation of a debt covenant is costly to the borrower (Beneish and Press, 1993), and the costs can be imposed on the borrower by various means such as imposition of additional constraints, an increase in interest rates, an increase in collateral, or requiring the immediate repayment of the debt (Gopalakrishnan and Parkash, 1995). In this paper, the costs imposed on the borrower after a violation of the debt covenant are reflected through an increased face value of debt. The initial face value of debt is denoted as F_n.

15 In this section our model is complementary to the model in Core and Schrand (1999). Both models examine the covenant effect and signalling effect of accounting information. In Core and Schrand (1999), a firm's accounting earnings in a period are the sum of cash-flow relevant earnings and noise, and the firm's accounting value is affected by its accumulated total accounting earnings. Their model focuses on the characteristics of earnings and cash-flows, where noise is exogenously determined. Our model explicitly considers the effect of noise in accounting information on firm valuation. Specifically, in our model accounting precision and bias are choice variables to the firm.

16 In practice, we often observe covenants that specify a minimum level of net worth (or net assets) that must be maintained. We view our minimum book value of assets, H, as analogous to net worth restrictions. For simplicity we employ the book value of assets as the critical value.

If the accounting value at the end of period one (S_1) is not lower than H, then the face value of debt remains at F_n. If the accounting value is lower than H, the face value of debt at the end of period two is increased to F_v ($F_v > F_n$). The difference between F_v and F_n represents the costs incurred by the borrower for violating the debt covenant. The preset level H can be interpreted as an accounting-based covenant level. Let $h = \ln \frac{H}{A_0}$.

Proposition 3 describes the relationship between equity value and the precision in accounting information when an interim review exists.

Proposition 3: When a debt covenant exists, the expected value of equity is an increasing function of the precision in accounting information for periods one and two.

Proof: See the Appendix.□

Since equity holders make a decision on whether to pay back debt at the end of period two, increasing the precision in accounting information for period two always increases the equity value by reducing the probability of making type I or type II errors. Increasing the precision in accounting information for period one may increase or decrease the probability that the debt covenant will be breached, depending on the level of debt and the debt covenant. However, for reasonable debt covenant levels,[17] the probability that the debt covenant will not be breached increases in the level of accounting precision. Equity value therefore increases in precision for reasonable debt covenant restrictions.

Equity holders may have an incentive to include a bias in the accounting value at the end of period one in order to avoid a potential debt covenant violation. If we assume that the likelihood of detection of the bias by auditors/analysts is an increasing function of the size of the bias and the accounting precision, then equity holders may have an incentive to reduce accounting precision for the period one report. Denote this likelihood as $p(b, \alpha)$. Then the expected value of the firm at the end of the second period is:

$$\begin{aligned} V_2 = {} & p(b, \alpha_1)[E(V_2|z_1 < h, F = F_v) + E(V_2|z_1 \geq h, F = F_n) - L] \\ & + (1 - p(b, \alpha_1))[E(V_2|z_1 < h - b, F = F_v) + E(V_2|z_1 \geq h - b, F = F_n)] \end{aligned} \tag{15}$$

where L is the cost to the firm if a bias is detected, which is an increasing function of b with $L(b = 0) = 0$.

A bias can reduce the probability of a potential violation of the debt covenant and its associated costs. If the cost to the firm resulting from a detected bias is low, equity holders will incorporate the bias in the accounting value. The accounting precision for period one can affect the bias decision. Specifically, accounting precision can have an impact in the following three ways: (1) higher precision increases the detection of an incorporated bias; (2) higher precision reduces the type one and type two errors with respect to debt covenant violation; and, (3) higher precision in period one can reduce the type one and type two errors for the period two debt payment decision. The benefit from increasing accounting precision will be lower if there is a bias.

17 We believe that the debt covenant (H) should not be higher than the firm's expected intrinsic value (A_0e^r). We regard all covenants lower than A_0e^r as reasonable.

3. A MODEL WHERE EQUITY HOLDERS HAVE PRIVATE INFORMATION ABOUT FIRM VALUE

When equity holders have private information about firm value, accounting precision is only relevant to equity holders if there is an interim review of debt covenants. We consider a two period model with an interim review in the following analysis.

Assume that at the end of period one equity holders observe the firm value y_1 and then they choose the accounting precision for the period. If the accounting value in the period is lower than h (the debt covenant level) there is a violation of the covenant. The firm's expected value at the end of the second period is (without bias incorporated):

$$V_2 = \Phi\left(\frac{h-y_1}{\alpha_1}\right)E(V_2|y_1, F = F_v)) + \left(1-\Phi\left(\frac{h-y_1}{\alpha_1}\right)\right)E(V_2|y_1, F = F_n). \quad (16)$$

Since equity holders can observe the true value of the firm, type I and type II errors with respect to the debt payment do not exist anymore.

If we allow the equity holder to incorporate bias in the financial report, then the firm's expected value is:

$$\begin{aligned} V_2 \ = \ & p(b,\alpha_1)\left[\Phi\left(\frac{h-y_1}{\alpha_1}\right)E(V_2|y_1, F=F_v))+\left(1-\Phi\left(\frac{h-y_1}{\alpha_1}\right)\right)E(V_2|y_1, F=F_n)-L\right] \\ & + (1-p(b,\alpha_1))\left[\Phi\left(\frac{h-b-y_1}{\alpha_1}\right)E(V_2|y_1, F=F_v))+\left(1-\Phi\left(\frac{h-b-y_1}{\alpha_1}\right)\right)E(V_2|y_1, F=F_n)\right]. \end{aligned} \quad (17)$$

Proposition 4: The accounting precision in period one is at its possible lowest level if $y_1 < h$. However, if $y_1 \geq h$, accounting precision will be the highest when firm value is at a moderately high level.

Proof: See the Appendix.□

When $y_1 > h$, the firm wants this message to be correctly conveyed to debt holders. To reduce the probability that the message is incorrectly conveyed the firm will want to improve accounting precision. If y_1 is slightly higher than h, improving accounting precision will not noticeably reduce the probability of a type I error. When y_1 is increased further, the marginal effect of higher precision on reducing the probability of a type I error becomes greater. As a result, accounting precision is an increasing function of firm value. When firm value is very high, the probability of a type I error is relatively low and the marginal benefit of increasing accounting precision is also low. As a result, accounting precision decreases as firm value becomes very high. Overall, we expect to see the best accounting systems from companies that perform reasonably well.

When bias is incorporated in the accounting report, it is most likely that the bias is incorporated when firm value is close to the debt covenant level. When the firm value is too high, the benefit from the bias is very limited. When the firm value is too low, the cost from bias is too high. Therefore, small to mid-sized errors should be observed with high frequency when firm value is close to the debt covenant level. A large bias is possible when firm value is low and the penalty for misreporting is not great.

Proposition 5: Bias is most likely to be incorporated into the accounting value when firm value is close to the debt covenant level.

Proof: See the Appendix.□

5. CONCLUSION

In this paper we examine the effect of financial leverage on shareholders' incentives to provide greater precision in accounting information. We combine a traditional equity valuation model developed in finance with periodic imperfect accounting information to derive expected equity value. We conclude that accounting precision is an increasing function of debt. If precision in accounting information for the second period is set after observing the accounting value at the end of period one, the precision set by equity holders for period two is highest if the observed accounting value at the end of period one is near the face value of debt.

When private information for equity holders exists, firms with moderately high performance will have the most accurate accounting information and firms with low performance will have the least accurate accounting information. Since the accounting precision of a firm is determined by the quality of its internal controls and the quality of its auditor, it is possible to empirically test the results derived in this paper. One could examine the relationship between the quality of a firm's internal controls and its auditor, and the face value of its debt and observed accounting value. We leave this as an avenue for future research.

APPENDIX

Proof of Lemma 1

At the end of the period, if the expected value of the firm conditional on the accounting value is not lower than the face value of debt, equity holders who want to maximize equity value will repay the debt. Otherwise, equity holders will leave the firm to the debt holders. The conditional expected value of the firm after observing the accounting value of the firm, S_1, is:

$$P_1 = E(A_1|S_1) = \int_{-\infty}^{\infty} A_0\, e^{y_1} f(y_1|z_1)\, dy_1 \tag{18}$$

where:

$$f(y_1|z_1) = \frac{1}{\sqrt{2\pi \dfrac{\sigma^2 \bullet \alpha_1^2}{\sigma^2+\alpha_1^2}}} \bullet e^{-\dfrac{\sigma^2+\alpha_1^2}{2\sigma^2 \bullet \alpha_1^2}\left(y_1 - \dfrac{\alpha_1^2\mu + \sigma^2 z_1 + \sigma^2\alpha_1^2/2}{\sigma^2+\alpha_1^2}\right)^2} \tag{19}$$

then:

$$\begin{aligned} P_1 &= \textstyle\int_{-\infty}^{\infty} A_0 e^{y_1} f(y_1|z_1)\, dy_1 \\ &= A_0 \bullet e^{\dfrac{\alpha_1^2\mu + (z_1+\alpha_1^2/2)\sigma^2}{\sigma^2+\alpha_1^2} + \dfrac{1}{2}\dfrac{\sigma^2 \bullet \alpha_1^2}{\sigma^2+\alpha_1^2}} \\ &= A_0 e^{r\dfrac{\alpha_1^2}{\sigma^2+\alpha_1^2}} \bullet e^{z_1\dfrac{\sigma^2}{\sigma^2+\alpha_1^2}} \bullet e^{\dfrac{\alpha_1^2}{2}\dfrac{\sigma^2}{\sigma^2+\alpha_1^2}}. \end{aligned} \tag{20}$$

Equity holders will repay the debt only if:

$$P_1 \geq F. \tag{21}$$

This implies:

$$z_1 \geq \frac{\alpha_1^2}{\sigma^2} \ln \frac{F}{A_0 e^r} + \ln \frac{F}{A_0} - \frac{\alpha_1^2}{2}. \tag{22}$$

Let:

$$z_1^* = \frac{\alpha_1^2}{\sigma^2} \ln \frac{F}{A_0 e^r} + \ln \frac{F}{A_0} - \frac{\alpha_1^2}{2}, \tag{23}$$

then:

$$S_1^* = A_0\, e^{z_1^*} = A_0^{-\frac{\alpha_1^2}{\sigma^2}} F^{1+\frac{\alpha_1^2}{\sigma^2}}\, e^{-r\frac{\alpha_1^2}{\sigma^2}-\frac{\alpha_1^2}{2}}. \tag{24}$$

Therefore, if $S_1 \geq S_1^*$, equity holders will repay the debt.□

Proof of Lemma 2

Given that equity holders will leave the firm to debt holders if $z_1 < z_1^*$, the expected value of equity is:

$$V_1 = e^{-r} \int_{z_1^*}^{\infty} f(z_1) \int_{-\infty}^{\infty} (A_0 e^{y_1} - F)\, f(y_1|z_1)\, dy_1 dz_1 = e^{-r} \int_{z_1^*}^{\infty} (P_1 - F)\, f(z_1)\, dz_1 \tag{25}$$

which can be simplified as:

$$V_1 = A_0 \Phi \left[\frac{\frac{\alpha_1^2}{\sigma^2} \ln \frac{A_0 e^r}{F} + \ln \frac{A_0}{F} + \mu + \sigma^2}{\sqrt{\alpha_1^2 + \sigma^2}} \right] - F e^{-r} \Phi \left[\frac{\frac{\alpha_1^2}{\sigma^2} \ln \frac{A_0 e^r}{F} + \ln \frac{A_0}{F} + \mu}{\sqrt{\alpha_1^2 + \sigma^2}} \right]. \quad \square \tag{26}$$

Proof of Proposition 1

Denote:

$$d_1 = \frac{\frac{\alpha_1^2}{\sigma^2} \ln \frac{A_0}{F e^{-r}} + \ln \frac{A_0}{F} + \mu + \sigma^2}{\sqrt{\sigma^2 + \alpha_1^2}} \tag{27}$$

and

$$g_1 = \frac{\sigma^2}{\sqrt{\sigma^2 + \alpha_1^2}}. \tag{28}$$

Then, from (26):

$$\begin{aligned}\frac{\partial V_1}{\partial \alpha_1} &= A_0\Phi'(d_1)\frac{\partial d_1}{\partial \alpha_1} - Fe^{-r}\Phi'[d_1-g_1]\left(\frac{\partial d_1}{\partial \alpha_1}-\frac{\partial g_1}{\partial \alpha_1}\right)\\ &= \Phi'[d_1]\frac{\partial d_1}{\partial \alpha_1}\left(A_0 - Fe^{-r+\frac{2\sigma^2\left(\frac{\alpha_1^2+\sigma^2}{\sigma^2}\ln\frac{A_0}{F}+\frac{\alpha_1^2}{\sigma^2}r+\mu+\sigma^2\right)-\sigma^4}{2(\sigma^2+\alpha_1^2)}}\right) + Fe^{-r}\Phi'[d_1-g_1]\frac{\partial g_1}{\partial \alpha_1}\\ &= \Phi'[d_1]\frac{\partial d_1}{\partial \alpha_1}A_0(1-e^0) - Fe^{-r}\Phi'[d_1-g_1]\frac{\alpha_1\sigma^2}{\left(\sigma^2+\alpha_1^2\right)^{3/2}}\\ &= -Fe^{-r}\Phi'[d_1-g_1]\frac{\alpha_1\sigma^2}{\left(\sigma^2+\alpha_1^2\right)^{3/2}} < 0. \end{aligned} \tag{29}$$

□

Proof of Corollary 1

$$\begin{aligned}\frac{\partial}{\partial F}\left(\frac{\partial V_1}{\partial \alpha_1}\right) &= -e^r\Phi'[d_1-g_1]\frac{\alpha_1\sigma^2}{(\sigma^2+\alpha_1^2)^{\frac{3}{2}}}\left(1+\frac{\sqrt{\sigma^2+\alpha_1^2}}{\sigma^2}(d_1-g_1)\right)\\ &= -e^r\Phi'[d_1-g_1]\frac{\alpha_1\sigma^2}{(\sigma^2+\alpha_1^2)^{\frac{3}{2}}}\left(1+\frac{1}{\sigma^2}\left(\frac{\sigma^2+\alpha_1^2}{\sigma^2}\ln\frac{A_0}{F}+\frac{\alpha_1^2}{\sigma^2}r+\mu\right)\right).\end{aligned} \tag{30}$$

From (30), $\frac{\partial}{\partial F}(\frac{\partial V_1}{\partial \alpha_1}) = 0$ when $F = F^* \equiv A_0 e^{r+\frac{\sigma^2}{2}\bullet\frac{\sigma^2}{(\sigma^2+\alpha_1^2)}}$, $\frac{\partial}{\partial F}(\frac{\partial V_1}{\partial \alpha_1}) < 0$ when $F < F^*$ and $\frac{\partial}{\partial F}\left(\frac{\partial V_1}{\partial \alpha_1}\right) > 0$ when $F > F^*$. Since the face value of the debt would not be greater than the expected firm value, then $F < F^*$ and the marginal benefit of increasing precision in accounting information is an increasing function of the face value of debt.□

Proof of Lemma 3

At the end of the second period, equity holders may choose to repay the debt or leave the firm to the debt holders. Since equity holders can only observe accounting information S_1 and S_2 at the end of periods one and two respectively, equity holders will compute the conditionally expected firm value, P_2, and compare it with the face value of debt in order to make a decision. The expected firm value given the observed firm value S_2 can be calculated as follows:

$$P_2 = E(A_2|S_1, S_2) = \int_{-\infty}^{\infty} A_0 e^{y_2} f(y_2|z_1, z_2)\, dy_2 \tag{31}$$

where the density function is:

$$f(y_2|z_1, z_2) = \frac{1}{\sqrt{2\pi\frac{\gamma^2\bullet\alpha_2^2}{\gamma^2+\alpha_2^2}}}\bullet e^{-\frac{\gamma^2+\alpha_2^2}{2\gamma^2\bullet\alpha_2^2}\left(y_2-\frac{\alpha_2^2 m+\gamma^2 z_2+\gamma^2\alpha_2^2/2)}{\gamma^2+\alpha_2^2}\right)^2} \tag{32}$$

where:

$$m = \mu + \frac{\alpha_1^2 \mu + \sigma^2 z_1 + \sigma^2 \frac{\alpha_1^2}{2}}{\sigma^2 + \alpha_1^2} \tag{33}$$

$$\gamma^2 = \frac{\sigma^2 (\sigma^2 + 2\alpha_1^2)}{\sigma^2 + \alpha_1^2}. \tag{34}$$

Then:

$$P_2 = \int_{-\infty}^{\infty} A_0 e^{y_2} f(y_2 | z_1, z_2)\, dy_2 = A_0 \bullet e^{\frac{\alpha_2^2 m + (z_2 + \alpha_2^2/2)\gamma^2}{\gamma^2 + \alpha_2^2} + \frac{1}{2} \frac{\gamma^2 \bullet \alpha_2^2}{\gamma^2 + \alpha_2^2}}. \tag{35}$$

Therefore, equity holders will pay back the debt only if:

$$z_2 \geq z_2^* = \frac{\gamma^2 + {\alpha_2}^2}{\gamma^2} \ln \frac{F}{A_0} - \frac{\alpha_2^2}{\gamma^2} m - \alpha_2^2. \tag{36}$$

Then, the threshold accounting value S_2^* has the following value:

$$S_2^* = A_0 e^{z_2^*} = A_0^{-\frac{\alpha_2^2}{\gamma^2}} F^{1 + \frac{\alpha_2^2}{\gamma^2}} e^{-m \frac{\alpha_2^2}{\gamma^2} - \alpha_2^2}. \quad \square \tag{37}$$

Proof of Lemma 4

$$V_2 = \int_{-\infty}^{\infty} E(V_2 | z_1, F)\, f(z_1)\, dz_1. \tag{38}$$

The expected value of equity at the end of period two given z_1 is:

$$\begin{aligned} E(V_2 | z_1, F) &= e^{-2r} \int_{z_2^*}^{\infty} f(z_2) \int_{-\infty}^{\infty} (A_0 e^{y_2} - F)\, f(y_2 | z_1, z_2)\, dy_2\, dz_2 \\ &= e^{-2r} \int_{z_2^*}^{\infty} (P_2 - F)\, f(z_2)\, dz_2 \end{aligned} \tag{39}$$

which can be simplified to:

$$E(V_2 | z_1, F) = e^{-2r} \left(A_0 e^{m + \frac{\gamma^2}{2}} \Phi \left[\frac{m + \gamma^2 - \frac{\alpha_2^2}{2} - z_2^*}{\sqrt{\gamma^2 + \alpha_2^2}} \right] - F \Phi \left[\frac{m - \frac{\alpha_2^2}{2} - z_2^*}{\sqrt{\gamma^2 + \alpha_2^2}} \right] \right). \tag{40}$$

Expanding (38):

$$V_2 = e^{-2r}\left(e^{2\mu+\sigma^2}\Phi\left[\frac{A+\frac{\gamma^2}{\sqrt{\gamma^2+\alpha_2^2}}+B\left(\mu-\frac{\alpha_1^2}{2}+\sigma^2\right)}{\sqrt{B^2\left(\sigma^2+\alpha_1^2\right)+1}}\right]-F\Phi\left[\frac{A+B\left(\mu-\frac{\alpha_1^2}{2}\right)}{\sqrt{B^2\left(\sigma^2+\alpha_1^2\right)+1}}\right]\right) \tag{41}$$

where:

$$A = \frac{\frac{\gamma^2+\alpha_2^2}{\gamma^2(\sigma^2+\alpha_1^2)}\left(\mu(\sigma^2+2\alpha_1^2)+\sigma^2\frac{\alpha_1^2}{2}\right)+\frac{\alpha_2^2}{2}-\frac{\gamma^2+\alpha_2^2}{\gamma^2}\ln\frac{F}{A_0}}{\sqrt{\gamma^2+\alpha_2^2}} \tag{42}$$

and

$$B = \frac{\sigma^2\left(\gamma^2+\alpha_2^2\right)}{\gamma^2\left(\sigma^2+\alpha_1^2\right)\sqrt{\gamma^2+\alpha_2^2}}. \tag{43}$$

Then:

$$V_2 = e^{-2r}\left(A_0 e^{2\mu+\sigma^2}\Phi[d_2]-F\Phi[d_2-C]\right) \tag{44}$$

where:

$$d_2 = \frac{A+\frac{\gamma^2}{\sqrt{\gamma^2+\alpha_2^2}}+B\left(\mu-\frac{\alpha_1^2}{2}+\sigma^2\right)}{\sqrt{B^2\left(\sigma^2+\alpha_1^2\right)+1}} \tag{45}$$

and

$$C = \frac{\frac{\gamma^2}{\sqrt{\gamma^2+\alpha_2^2}}+B\sigma^2}{\sqrt{B^2\left(\sigma^2+\alpha_1^2\right)+1}} = \sqrt{\frac{\sigma^4}{\left(\sigma^2+\alpha_1^2\right)}+\frac{\gamma^4}{\left(\gamma^2+\alpha_2^2\right)}}. \quad \square \tag{46}$$

Proof of Proposition 2

From (44):

$$\begin{aligned}
\frac{\partial V_2}{\partial\alpha_2} &= e^{-2r}\left(A_0 e^{2\mu+\sigma^2}\Phi'[d]\frac{\partial d_2}{\partial\alpha_2}-F\Phi'[d_2-C]\left(\frac{\partial d_2}{\partial\alpha_2}-\frac{\partial C}{\partial\alpha_2}\right)\right)\\
&= e^{-2r}\left(\Phi'[d_2]\frac{\partial d_2}{\partial\alpha_2}\left(A_0 e^{2\mu+\sigma^2}-F e^{d_2 C-\frac{C^2}{2}}\right)+F\Phi'[d_2-C]\frac{\partial C}{\partial\alpha_2}\right)\\
&= e^{-2r}\left(\Phi'[d_2]\frac{\partial d_2}{\partial\alpha_2}A_0 e^{2\mu+\sigma^2}(1-1)+F\Phi'[d_2-C]\frac{\partial C}{\partial\alpha_2}\right)\\
&= e^{-2r}\left(F\Phi'[d_2-C]\frac{\partial C}{\partial\alpha_2}\right) < 0.
\end{aligned} \tag{47}$$

As from (46):

$$\frac{\partial C}{\partial \alpha_2} = -\frac{\alpha_2 \gamma^4}{C\left(\gamma^2 + \alpha_2^2\right)^2} < 0. \tag{48}$$

Similarly, from (44):

$$\begin{aligned}
\frac{\partial V_2}{\partial \alpha_1} &= e^{-2r}\left(A_0 e^{2\mu+\sigma^2}\Phi'[d]\frac{\partial d_2}{\partial \alpha_1} - F\Phi'[d_2 - C]\left(\frac{\partial d_2}{\partial \alpha_1} - \frac{\partial C}{\partial \alpha_1}\right)\right)\\
&= e^{-2r}\left(\Phi'[d_2]\frac{\partial d_2}{\partial \alpha_1}\left(A_0 e^{2\mu+\sigma^2} - F e^{d_2 C - \frac{C^2}{2}}\right) + F\Phi'[d_2 - C]\frac{\partial C}{\partial \alpha_1}\right)\\
&= e^{-2r}\left(\Phi'[d_2]\frac{\partial d_2}{\partial \alpha_1} A_0 e^{2\mu+\sigma^2}(1-1) + F\Phi'[d_2 - C]\frac{\partial C}{\partial \alpha_1}\right)\\
&= e^{-2r}\left(F\Phi'[d_2 - C]\frac{\partial C}{\partial \alpha_1}\right) < 0.
\end{aligned} \tag{49}$$

As from (46):

$$\frac{\partial C}{\partial \alpha_1} = -\frac{\alpha_1 \sigma^4 \alpha_2^4}{C\left(\sigma^2 + \alpha_1^2\right)^2\left(\gamma^2 + \alpha_2^2\right)^2} < 0. \tag{50}$$

Proof of Corollary 2

The ratio of the marginal benefit obtained by increasing accounting precision for period two and period one is:

$$\frac{\frac{\partial V_2}{\partial \alpha_2}}{\frac{\partial V_2}{\partial \alpha_1}} = \frac{\frac{\partial C}{\partial \alpha_2}}{\frac{\partial C}{\partial \alpha_1}} = 4\frac{\alpha_1^3}{\alpha_2^3} + 4\frac{\sigma^2 \alpha_1}{\alpha_2^3} + \frac{\sigma^4}{\alpha_1 \alpha_2^3}. \tag{51}$$

If $4\alpha_1^4 + 4\alpha_1^2\,\sigma^2 + \sigma^4 - \alpha_1\,\alpha_2^3 > 0$, the value of equation (51) is greater than one. Therefore, if accounting precision in the two periods are similar, then the benefit from increasing accounting precision in period two is greater than the benefit from increasing accounting precision in period one.□

Proof of Corollary 3

After observing the accounting value at the end of period one, the expected equity value at the end of period two is:

$$E(V_2|z_1, F) = e^{-2r}\left(A_0 e^{m+\frac{\gamma^2}{2}}\Phi\left[\frac{m + \gamma^2 - \frac{\alpha_2^2}{2} - z_2^*}{\sqrt{\gamma^2 + \alpha_2^2}}\right] - F\Phi\left[\frac{m - \frac{\alpha_2^2}{2} - z_2^*}{\sqrt{\gamma^2 + \alpha_2^2}}\right]\right). \tag{52}$$

Then:

$$\begin{aligned}
\frac{\partial E(V_2|z_1, F)}{\partial \alpha_2} &= A_0 e^{m+\frac{\gamma^2}{2}} \Phi'(d) \frac{\partial d_{2z}}{\partial \alpha_2} - F\Phi'[d_{2z} - g_2] \left(\frac{\partial d_{2z}}{\partial \alpha_2} - \frac{\partial g_2}{\partial \alpha_2} \right) \\
&= \Phi'[d_{2z}] \frac{\partial d_{2z}}{\partial \alpha_2} \left(A_0 e^{m+\frac{\gamma^2}{2}} - F\, e^{\frac{2\gamma^2 \left(\frac{\gamma^2+\alpha_2^2}{\gamma^2} \left(m + \ln \frac{A_0}{F} + \gamma^2 + \alpha_2^2 \right) - \gamma^4 \right)}{2(\gamma^2+\alpha^2)}} \right) + F\Phi'[d_{2z} - g_2] \frac{\partial g_2}{\partial \alpha_2} \\
&= \Phi'[d_{2z}] \frac{\partial d_{2z}}{\partial \alpha_2} A_0 e^{m+\frac{\gamma^2}{2}} (1-1) - F\Phi'[d_{2z} - g_2] \frac{\alpha_2 \gamma^2}{(\gamma^2 + \alpha_2^2)^{3/2}} \\
&= -F\Phi'[d_{2z} - g_2] \frac{\alpha_2 \gamma^2}{(\gamma^2 + \alpha_2^2)^{3/2}} < 0
\end{aligned} \tag{53}$$

where:

$$d_{2z} = \frac{m + \gamma^2 - \frac{\alpha_2^2}{2} - z_2^*}{\sqrt{\gamma^2 + \alpha_2^2}} \tag{54}$$

$$g_2 = \frac{\gamma^2}{\sqrt{\gamma^2 + \alpha_2^2}}. \tag{55}$$

Next we consider under what value of z_1 the accounting precision in the second period has the highest marginal value:

$$\begin{aligned}
\frac{\partial^2 E(V_2|z_1, F)}{\partial z_1 \partial \alpha_2} &= -F \frac{\partial \Phi'[d_{2z} - g_2]}{\partial z_1} \frac{\alpha_2 \gamma^2}{(\gamma^2 + \alpha_2^2)^{3/2}} \\
&= \frac{F\Phi'[d_{2z} - g_2]\sigma^2 \alpha_2^2}{(\sigma^2 + \alpha_1^2)(\gamma^2 + \alpha_2^2)^{3/2}} \left(\frac{\gamma^2 + \alpha_2^2}{\gamma^2} \left(\frac{(\sigma^2 + 2\alpha_1^2)\mu + \sigma^2 z_1 + \sigma^2 \frac{\alpha_1^2}{2}}{\sigma^2 + \alpha_1^2} - \ln \frac{F}{A_0} \right) + \frac{\alpha_2^2}{2} \right)
\end{aligned} \tag{56}$$

if:

$$\frac{\gamma^2 + \alpha_2^2}{\gamma^2} \left(\frac{(\sigma^2 + 2\alpha_1^2)\mu + \sigma^2 z_1 + \sigma^2 \frac{\alpha_1^2}{2}}{\sigma^2 + \alpha_1^2} - \ln \frac{F}{A_0} \right) + \frac{\alpha_2^2}{2} > 0 \tag{57}$$

or equivalently:

$$z_1 > \left(\frac{\sigma^2 + \alpha_1^2}{\sigma^2} \left(\ln \frac{F}{A_0} - \frac{\gamma^2 \alpha_2^2}{2(\gamma^2 + \alpha_2^2)} \right) - \frac{(\sigma^2 + 2\alpha_1^2)\mu}{\sigma^2} - \frac{\alpha_1^2}{2} \right),$$

then the marginal benefit of increasing accounting precision is a decreasing function of the observed accounting value at the end of period one. Note that the threshold value of z_1 is a function of the accounting precision in the second period. Then for a given cost function to achieve a certain accounting precision in the second period, there exists an optimal value of α_2 for every observed value z_1. Let:

$$z_1^* = \left(\frac{\sigma^2 + \alpha_1^2}{\sigma^2}\left(\ln\frac{F}{A_0} - \frac{\gamma^2\alpha_2^{*2}}{2(\gamma^2 + \alpha_2^{*2})}\right) - \frac{(\sigma^2 + 2\alpha_1^2)\mu}{\sigma^2} - \frac{\alpha_1^2}{2}\right) \tag{58}$$

where $\alpha_2{}^*$ is the optimal accounting precision in the second period after observing z_1^*. Then z_1^* is the accounting value in the first period which results in the highest incremental value for the accounting precision in the second period.□

Proof of Proposition 3

From equation (40), with a given face value of debt and observed accounting value z_1, the firm's expected value is:

$$E(V_2|z_1, F) = e^{-2r}\left(A_0 e^{m+\frac{\gamma^2}{2}}\Phi\left[\frac{m + \gamma^2 - \frac{\alpha_2^2}{2} - z_{2c}^*}{\sqrt{\gamma^2 + \alpha_2^2}}\right] - F\Phi\left[\frac{m - \frac{\alpha_2^2}{2} - z_{2c}^*}{\sqrt{\gamma^2 + \alpha_2^2}}\right]\right). \tag{59}$$

When debt covenants exist, the face value of debt is F_v if the covenant is violated, and the face value is F_n if the covenant is not violated. The cut off value z_{2c}^* is a function of the face value of debt in the second period. Therefore:

$$z_{2c}^* = z_{2n}^* = \frac{\gamma^2 + {\alpha_2}^2}{\gamma^2}\ln\frac{F_n}{A_0} - \frac{\alpha_2^2}{\gamma^2}m - \alpha_2^2 \quad \text{if} \quad F = F_n \tag{60}$$

$$z_{2c}^* = z_{2v}^* = \frac{\gamma^2 + {\alpha_2}^2}{\gamma^2}\ln\frac{F_v}{A_0} - \frac{\alpha_2^2}{\gamma^2}m - \alpha_2^2 \quad \text{if} \quad F = F_v. \tag{61}$$

The expected value of the firm with a debt covenant at the end of the second period is therefore:

$$\begin{aligned} V_2 &= E(V_2|z_1 < h,\ F = F_v) + E(V_2|z_1 \geq h,\ F = F_n) \\ &= \int_{-\infty}^{h} E(V_2|z_1, F = F_v)\, f(z_1)\, dz_1 + \int_{h}^{\infty} E(V_2|z_1,\ F = F_n)\, f(z_1)\, dz_1. \end{aligned} \tag{62}$$

Denote:

$$d_{2n} = \frac{m + \gamma^2 - \frac{\alpha_2^2}{2} - z_{2n}^*}{\sqrt{\gamma^2 + \alpha_2^2}} \tag{63}$$

$$d_{2v} = \frac{m + \gamma^2 - \frac{\alpha_2^2}{2} - z_{2v}^*}{\sqrt{\gamma^2 + \alpha_2^2}}. \tag{64}$$

Since:

$$\frac{\partial E(V_2|z_1, F)}{\partial \alpha_2} = -F\Phi'[d_{2c} - g_2]\frac{\alpha_2\gamma^2}{(\gamma^2 + \alpha_2^2)^{3/2}} < 0 \tag{65}$$

and $f(z_1) > 0$, then:

$$\frac{\partial V_2}{\partial \alpha_2} = \int_{-\infty}^{h} \frac{\partial E(V_2|z_1, F = F_v)}{\partial \alpha_2} f(z_1)\, dz_1 + \int_{h}^{\infty} \frac{\partial E(V_2|z_1, F = F_n)}{\partial \alpha_2} f(z_1)\, dz_1 < 0 \tag{66}$$

where d_{2c} represents either d_{2v} or d_{2n}.

Next we consider the relationship between expected value and precision in period one:

$$\begin{aligned}\frac{\partial V_2}{\partial \alpha_1} &= \int_{-\infty}^{h} \frac{\partial E(V_2|z_1, F = F_v)}{\partial \alpha_1} f(z_1)\, dz_1 + \int_{h}^{\infty} \frac{\partial E(V_2|z_1, F = F_n)}{\partial \alpha_1} f(z_1)\, dz_1 \\ &\quad + \int_{-\infty}^{h} E(V_2|z_1, F = F_v)\frac{\partial f(z_1)}{\partial \alpha_1}\, dz_1 + \int_{h}^{\infty} E(V_2|z_1, F = F_n)\frac{\partial f(z_1)}{\partial \alpha_1}\, dz_1.\end{aligned} \tag{67}$$

The sum of the first two components on the right hand side of (67) is negative since accounting precision for period one affects the type I and type II error rates in respect to the debt payback decision at the end of period two. The sum of the last two components is negative if $H < A_0e^r$ (or $h < r$). If $H < A_0e^r$, the expected probability that the debt covenant is violated is a decreasing function of the accounting precision for period one, and if $H > A_0e^r$, the expected probability that the debt covenant is violated is an increasing function of the accounting precision for period one. Since A_0e^r is the expected firm value at the end of period one, realistically the debt covenant level will be lower than the expected firm value. As a result, increasing accounting precision for period one increases the expected equity value through reducing the expected probability that the debt covenant is violated. Therefore, it can be concluded that the expected equity value is an increasing function of the accounting precision for period one.

Proof of Proposition 4

$$V_2 = \Phi\left(\frac{h - y_1}{\alpha_1}\right) E(V_2|y_1, F = F_v)) + \left(1 - \Phi\left(\frac{h - y_1}{\alpha_1}\right)\right) E(V_2|y_1, F = F_n) \tag{68}$$

then:

$$\frac{\partial V_2}{\partial \alpha_1} = \Phi'\left(\frac{h - y_1}{\alpha_1}\right)\frac{h - y_1}{\alpha_1^2}[E(V_2|y_1, F = F_n)) - E(V_2|y_1, F = F_v)]. \tag{69}$$

Therefore, if $y_1 > h$, $\partial V/\partial\, \alpha_1 < 0$, if $y_1 = h$, $\partial V/\partial\, \alpha_1 = 0$ and if $y_1 < h$, $\partial V/\partial\, \alpha_1 > 0$.

Let a_1^* be the optimal level of variance for the accounting information for a given h and $y_1 > h$, then:

$$\frac{d\alpha_1^*}{dh} = -\frac{\Phi'\left(\frac{h-y_1}{\alpha_1^*}\right)\left(\frac{-(h-y_1)^2}{\alpha_1^{*4}} + \frac{1}{\alpha_1^{*2}}\right)}{\Phi'\left(\frac{h-y_1}{\alpha_1^*}\right)\left(\frac{(h-y_1)^3}{\alpha_1^{*5}} + \frac{-2(h-y)}{\alpha_1^{*3}}\right)} = \frac{(h-y_1)^2 - \alpha_1^{*2}}{(h-y)\alpha_1^*[(h-y_1)^2 - 2\alpha_1^{*2}]}. \quad (70)$$

If h is very small, $d\alpha_1^*/dh < 0$; if h is very close to y_1, $da_1^*/dh < 0$; if h is in the middle, then $da_1^*/dh > 0$. Therefore, a firm has the highest precision when the distance between y_1 and h is moderate.□

Proof of Proposition 5

When a bias is incorporated in the accounting value, then the expected firm value is:

$$V_2 = p(b,\alpha_1)\left[\Phi\left(\frac{h-y_1}{\alpha_1}\right)E(V_2|y_1, F=F_v)) + \left(1-\Phi\left(\frac{h-y_1}{\alpha_1}\right)\right)E(V_2|y_1, F=F_n) - L\right]$$
$$+ (1-p(b,\alpha_1))[\Phi\left(\frac{h-b-y_1}{\alpha_1}\right)E(V_2|y_1, F=F_v)) + \left(1-\Phi\left(\frac{h-b-y_1}{\alpha_1}\right)\right)E(V_2|y_1, F=F_n)]. \quad (71)$$

Then:

$$\frac{\partial V_2}{\partial b} = \frac{\partial p}{\partial b}\left[\left(\Phi\left(\frac{h-b-y_1}{\alpha_1}\right) - \Phi\left(\frac{h-y_1}{\alpha_1}\right)\right)(E(V_2|y_1, F=F_n) - E(V_2|y_1, F=F_v)) - L\right]$$
$$- P\frac{\partial L}{\partial b} + (1-p(b,\alpha_1))\left[\Phi'\left(\frac{h-b-y_1}{\alpha_1}\right)\frac{1}{\alpha_1}(E(V_2|y_1, F=F_n) - E(V_2|y_1, F=F_v))\right]. \quad (72)$$

The marginal benefit from a zero bias is:

$$\frac{\partial V_2(b=0)}{\partial b} = -\frac{\partial p}{\partial b}L + \Phi'\left(\frac{h-y_1}{\alpha_1}\right)\frac{1}{\alpha_1}[E(V_2|y_1, F=F_n) - E(V_2|y_1, F=F_v)]. \quad (73)$$

The maximum marginal value from a bias is obtained at $y_1 = h$.□

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