Exam #3
ECO 3401
Spring 2006
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NAME Key

Directions: Write solutions to the following problems legibly on this exam. Show all work to receive full credit. If you set up the solution to the problem correctly, you do not have to simplify the solution for full credit. There is a total of 100 points possible on this exam.

1. (8 points) Using the limit of the difference quotient

$$\lim_{h \to 0} \frac{[f(x+h) - f(x)]}{h}$$

Find the derivative of $y = 10x^2 + 4$

$y = 10x^2 + 4$

$f(x) = 10x^2 + 4$

$f(x+h) = 10(x+h)^2 + 4$

$= 10(x^2 + 2xh + h^2) + 4$

$= 10x^2 + 20xh + 10h^2 + 4$

$\frac{[f(x+h) - f(x)]}{h} = \frac{\cancel{10x^2} + 20xh + 10h^2 \cancel{+4} - \cancel{10x^2} - \cancel{4}}{h}$

$= \frac{20xh + 10h^2}{h}$

$= \frac{\cancel{h}[20x + 10h]}{\cancel{h}}$

$= 20x + 10h$

as $h \to 0$, $= 20x$

2. (8 points) At the Summa Corporation, Nadine Henley studied the relationship between income and number of years of work experience for Summa's new hires. She found that a person with x years of work experience before seeking employment with Summa can expect to receive an average yearly income of y dollars per year, where

$$y = 200x^{3/2} + 18{,}700$$

Find the rate of change of income with respect to number of years of work experience. Evaluate this expression when $x = 9$.

$\frac{\partial y}{\partial x} = 300x^{1/2}$

If $x = 9$, then $300(9)^{1/2} = 300(3) = \$900$

3. Suppose the total cost function for producing the popular TV show *Hullabaloo* is

$$c = 4 + q^2$$

a) (4 points) Find the marginal cost function. What is the value of marginal cost when $q = 2$?

$$MC = \frac{\partial c}{\partial q} = 2q, \quad q = 2 \Rightarrow 4$$

b) (4 points) Find the average cost function. What is the slope of the tangent line to the average cost function at the point $q = 2$?

$$AC = \frac{c}{q} = \frac{4 + q^2}{q} = \frac{4}{q} + q = 4q^{-1} + q$$

$$\frac{\partial AC}{\partial q} = -4q^{-2} \quad \text{if } q = 2 \Rightarrow -4(2)^{-2} = -1$$

c) (4 points) Find the equation for the line tangent to the average cost curve when $q = 3$.

$$y = mx + b \qquad m = -4(3)^{-2} = \frac{-4}{9}$$

when $q = 3$, $c = 4 + (3)^2$

$c = 4 + 9$

$c = 13$

$$13 = -\frac{4}{9}(3) + b$$

$$13 = -\frac{4}{3} + b$$

$$\frac{43}{3} = b, \text{ so} \quad c = -\frac{4}{9}q + \frac{43}{3}$$

4. (8 points) The total cost c of producing q units of Halo Shampoo is given by

$$c = 2{,}500 + 15q + 0.3q^3$$

If the price per unit p is given by the equation

$$q = 800 - 1.7p^2$$

Find the rate of change of cost with respect to price per unit when $p = 100$.

$$\frac{dc}{dp} = \left(\frac{dc}{dq}\right)\left(\frac{dq}{dp}\right) \quad \text{Chain rule}$$

$$\frac{dc}{dp} = [15 + 0.9q^2][-3.4p]$$

$$= -51p - 3.06pq^2$$

when $p = 100$, $q = 800 - 1.7(100)^2 = -16{,}200$

$$= -51(100) - 3.06(100)(-16{,}200)^2$$

5. (8 points) If the total-cost function for manufacturing new Cadillacs is given by

$$c = \frac{7q^2}{\sqrt{q^2+5}} + 3{,}000$$

Find the marginal-cost function.

$$c = \frac{7q^2 + 3000(q^2+5)^{1/2}}{(q^2+5)^{1/2}}$$

Use Quotient Rule:

$$\frac{dc}{dq} = \frac{(q^2+5)^{1/2}\left[14q + 1500(q^2+5)^{-1/2}(2q)\right] - \left[7q^2 + 3000(q^2+5)^{1/2}\right]\left[\tfrac{1}{2}(q^2+5)^{-1/2}(2q)\right]}{[q^2+5]}$$

$$\frac{dc}{dq} = \frac{14q(q^2+5)^{1/2} + \cancel{3000q} - 7q^3(q^2+5)^{-1/2} - \cancel{3000q}}{(q^2+5)}$$

$$\frac{dc}{dq} = \frac{14q(q^2+5)^{1/2} - 7q^3(q^2+5)^{-1/2}}{(q^2+5)} \cdot \frac{(q^2+5)^{1/2}}{(q^2+5)^{1/2}} \quad \leftarrow \text{multiply by 1!}$$

$$= \frac{14q(q^2+5) - 7q^3}{(q^2+5)^{3/2}} = \frac{14q^3 + 70q - 7q^3}{(q^2+5)^{3/2}} = \frac{7q(q^2+10)}{(q^2+5)^{3/2}}$$

6. (8 points) Find the first and second derivatives of

$$y = -5x^2 + 2x - 9$$

Does this function attain a maximum or minimum point? At what value of x?

$y' = -10x + 2$

$y'' = -10$

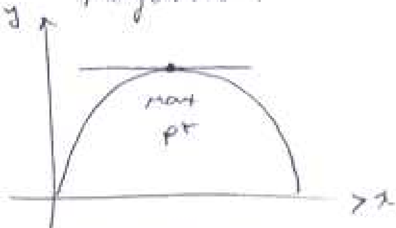
Concave Function b/c Second derivative is negative.

$-10x + 2 = 0$

$-10x = -2$

$x = \frac{2}{10}$

$x = \frac{1}{5}$



7. At Clark's, the manufacturer of Teaberry gum, the revenue r obtained from the sale of q packs of gum is given by

$$r = 50q - 0.5q^2$$

a) (4 points) Find the marginal revenue.

$MR = \frac{dr}{dq} = 50 - q$

b) (4 points) When $q = 10$, find the percentage rate of change of r.

Rate of change $= \frac{r'}{r}$

$\frac{r'}{r} = \frac{50 - q}{50q - 0.5q^2}$

at $q = 10$: $\frac{50 - 10}{0.5(10) - 0.5(100)} = \frac{40}{500 - 50} = 0.089$

8. Suppose the demand function for a Vikki Carr hit CD is $p = 500 - 2q$.

a) (4 points) Find the total revenue function. Find the equation of the tangent line to the total revenue function when $q = 100$. What information does this tangent line provide about the total revenue function?

$TR = p \cdot q = 500q - 2q^2$

$\frac{dTR}{dq} = 500 - 4q = m$

@ 100, $m = 500 - 400 = 100$

when $q = 100$, $TR = 500(100) - 2(100)^2 = 30{,}000$

$30{,}000 = 100(100) + b$

$20{,}000 = b$

$TR = 100q + 20{,}000$

b) (4 points) What is marginal revenue when $q = 100,\ 125,\ 150$?

$q = 100 \Rightarrow MR = 500 - 400 = 100$

$q = 125 \Rightarrow MR = 500 - 500 = 0$

$q = 150 \Rightarrow MR = 500 - 600 = -100$

c) (4 points) Use the first and second derivative tests to find the value of q that maximizes total revenue.

$$\frac{dTR}{dq} = 500 - 4q = 0$$

$$-4q = -500$$

$$q = 125$$

$$\frac{d^2TR}{dq^2} = -4 < 0$$

Concave function, so $q = 125$ is a max.

d) (4 points) Compute the price elasticity of demand when $q = 100,\ 125,\ 150$.

$\frac{dq}{dp} \cdot \frac{p}{q}$:

$p = 500 - 2q$

$2q = 500 - p$

$q = 250 - \frac{1}{2}p$

$\frac{dq}{dp} = -\frac{1}{2}$

If $q = 100$, $p = 300$

$-\frac{1}{2} \cdot \frac{300}{100} = -1.50$

If $q = 125$, $p = 250$

$-\frac{1}{2} \cdot \frac{250}{125} = -1.00$

If $q = 150$, $p = 200$

$-\frac{1}{2} \cdot \frac{200}{150} = -0.67$

$= -2/3$.

9. Differentiate the following functions with respect to x.

a) (4 points) $y = \dfrac{1}{x + \dfrac{1}{x+1}}$ $= \dfrac{1}{\dfrac{x(x+1)+1}{x+1}} = \dfrac{x+1}{x^2+x+1}$

$$\frac{dy}{dx} = \frac{(x^2+x+1)(1) - (x+1)(2x+1)}{(x^2+x+1)^2}$$

$$= \frac{(x^2+x+1) - (2x^2+3x+1)}{(x^2+x+1)^2}$$

$$= \frac{-x^2-2x}{(x^2+x+1)^2} = -\frac{x^2+2x}{(x^2+x+1)^2}$$

b) (4 points) $y = \frac{3}{2}\left(5\sqrt{p-2}\right)(3p-1)$

$$y = \frac{3}{2}\left(5(p-2)^{1/2}\right)(3p-1)$$

$$y = \left[\frac{15}{2}(p-2)^{1/2}\right]\left[3p-1\right]$$

$$y' = \left[\frac{15}{2}(p-2)^{1/2}\right][3] + [3p-1]\left[\frac{15}{4}(p-2)^{-1/2}\right]$$

$$y' = \frac{45(p-2)^{1/2}}{2} + \frac{(3p-1)(15)}{4(p-2)^{1/2}}$$

$$y' = \frac{75-45p}{4(p-2)^{1/2}}$$

c) (4 points) $y = \dfrac{x-5}{(x+2)(x-4)}$

$$y' = \frac{[(x+2)(x-4)](1) - (x-5)(2x-2)}{[(x+2)(x-4)]^2}$$

$$y' = \frac{x^2-2x-8-(2x^2-12x+10)}{[(x+2)(x-4)]^2}$$

$$y' = \frac{-(x^2-10x+18)}{[(x+2)(x-4)]^2}$$

10. Differentiate the following functions with respect to x.

a) (4 points) $y = \left(\frac{x-7}{x+4}\right)^{10}$

$$y' = 10\left[\frac{x-7}{x+4}\right]^9\left[\frac{(x+4)(1)-(x-7)(1)}{(x+4)^2}\right]$$

$$= 10\left[\frac{x-7}{x+4}\right]^9\left[\frac{11}{(x+4)^2}\right]$$

$$= \frac{110(x-7)^9}{(x+4)^{11}}$$

b) (4 points) $y = (8x-1)^3(2x+1)^4$

$$y' = (8x-1)^3\left[4(2x+1)^3(2)\right] + (2x+1)^4\left[3(8x-1)^2(8)\right]$$

$$= 8(8x-1)^2(2x+1)^3\left[(8x-1)+3(2x+1)\right]$$

$$= 8(8x-1)^2(2x+1)^3(14x+2)$$

$$= 16(8x-1)^2(2x+1)^3(7x+1)$$

c) (4 points) $y = (x^2-3)^{-2}$

$$y' = -2(x^2-3)^{-3}(2x)$$

$$= -4x(x^2-3)^{-3}$$