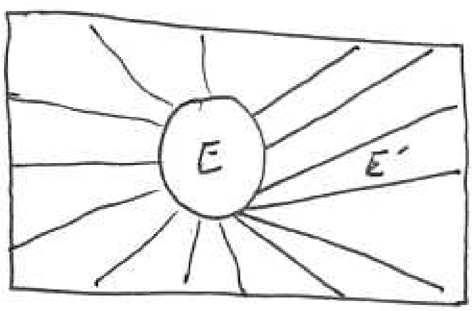
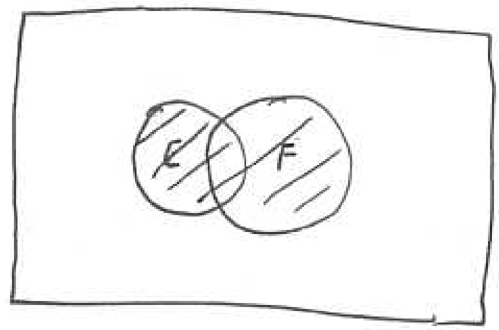
Complement, Union, Intersection

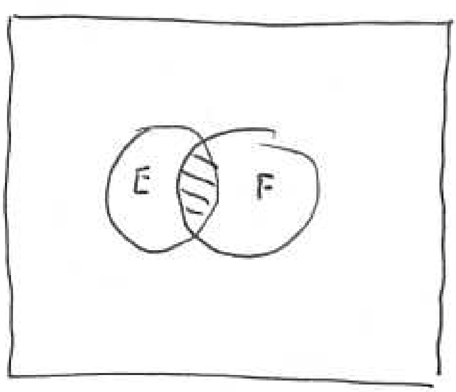
Definition: Let S be a Sample space for an experiment with events E and F. The <u>Complement</u> of E, denoted E', is the event consisting of all Sample points in S that are not in E. The <u>union</u> of E and F, denoted $E \cup F$ is the event consisting of all Sample points that are either in E or F, or in both E and F. The intersection of E and F, denoted $E \cap F$, is the event consisting of all sample points common to both E and F.



(a) Complement



(b) Union



(c) Intersection

Examples

1. Roll one die one time. Sample space would be

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let events be defined as

$$E = \{1, 3, 5\} \qquad F = \{3, 4, 5, 6\} \qquad G = \{1\}$$

Find

a. E' ; $E' = \{2, 4, 6\}$

b. $E \cup F$; $E \cup F = \{1, 3, 4, 5, 6\}$

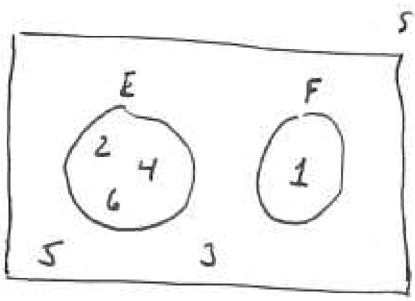
c. $E \cap F$; $E \cap F = \{3, 5\}$

d. $F \cap G$; $F \cap G = \emptyset$ "empty set"

e. $E \cup E'$; $E \cup E' = \{1, 2, 3, 4, 5, 6\} = S$

f. $E \cap E'$; $\emptyset$

Definition: Events E and F are mutually exclusive events if and only if $E \cap F = \emptyset$



Homework: pp. 409–410 #3, 7, 9, 11, 13, 15, 17, 27, 31

Probability

Definition: A sample space S is called an equiprobable sample space if and only if all of the events are equally likely to occur

Examples: Tossing a coin; rolling one die

Definition: If S is an equiprobable sample space with N sample points (or outcomes) $s_1, s_2, \ldots, s_N$ then the probability of the simple event $\{s_i\}$ is

$$P(s_i) = \frac{1}{N} \qquad \text{for } i = 1, 2, \ldots, N$$

Definition: If S is a finite equiprobable space for an experiment and $E = \{s_1, s_2, \ldots, s_j\}$ is an event then the probability of E is given by

$$P(E) = P(s_1) + P(s_2) + \cdots + P(s_j)$$

OR

$$P(E) = \frac{\#(E)}{\#(S)}$$

where $\#(E)$ = number of outcomes in E and $\#(S)$ = number of outcomes in S.

Examples:

1. Two coins are tossed. Determine the probability of

   a. two heads: $\frac{\#(E)}{\#(S)} = \frac{1}{4}$

   b. at least one head: $\frac{\#(E)}{\#(S)} = \frac{3}{4}$

   Note: $S = \{HH, HT, TH, TT\}$

Example 2: From an ordinary deck of 52 playing cards, two cards are randomly drawn without replacement. If E is the event that one of the cards is a 2 and the other is a 3, find $P(E)$

$$P(E) = \frac{\#(E)}{\#(S)} = \frac{4 \times 4}{{}_{52}C_2} = \frac{16}{1326} = \frac{8}{663}$$

Note: order does not matter, 4 suits, so 4 ways of drawing a 2 and 4 ways of drawing a 3. Sample space determined by the number of ways to draw 2 cards out of 52 (when order does not matter!).

Example 3: Find the probability of drawing four of a kind (like 4 kings) in a poker hand

$$P(E) = \frac{\#(E)}{\#(S)} = \frac{13 \times 48}{{}_{52}C_5} = \frac{13 \times 48}{2{,}598{,}960} \approx 0.00024$$

Note: A deck has 4 suits with 13 denominations. So 4 cards of one denomination can be drawn in 13 ways. 48 ways of selecting the last card.

Example 4: From a committee of three males and four females, a subcommittee of four persons is randomly selected. Find the probability that it consists of two males and two females

$$P(E) = \frac{{}_3C_2 \times {}_4C_2}{{}_7C_4} = \frac{\frac{3!}{2!\,1!} \times \frac{4!}{2!\,2!}}{\frac{7!}{4!\,3!}} = \frac{3 \times 6}{\frac{7 \cdot 6 \cdot 5}{6}} = \frac{18}{35}$$

Properties of probability: We know that.

$$0 \leq \frac{\#(E)}{\#(S)} \leq \frac{N}{N}$$

Thus, $0 \leq P(E) \leq 1$

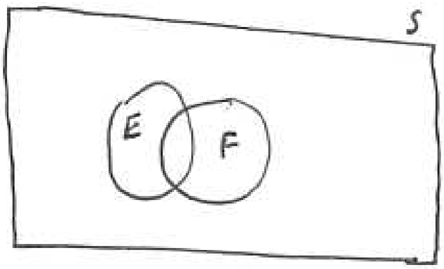
$$P(\emptyset) = 0$$

$$P(S) = 1$$

$$P(E) = 1 - P(E')$$

Probability of a union of events: If E and F are events, then

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$



Need to remove the sample points that are common to E and F, otherwise would double count.

Note if E and F are mutually exclusive events then $P(E \cap F) = \emptyset$

more examples:

1. From a production run of 5000 light bulbs, 2% of which are defective, one bulb is selected at random. What is the probability that the bulb is defective?

$$P(E) = \frac{\#(E)}{\#(S)} = \frac{100}{5000} = \frac{1}{50} = 0.02$$

Probability of bulb not defective

$$P(E') = 1 - 0.02 = 0.98$$

2. A pair of dice is rolled. what is the probability that the Sum of numbers that turn up is (a) 7 (b) 7 or 11 (c) greater than 3

a) $E_7 = \{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

$P_r(7) = 1/6 = 6/36$

b) $E_{11} = \{(6,5),(5,6)\}$

$P_r(11) = 2/36 = 1/18$

$P_r(7 \text{ or } 11) = P_r(7) + P_r(11) = 8/36 = 2/9$

(c) $P_r(2) = 1/36$

$P_r(3) = 2/36$

$P_r(\text{greater than } 3) = 33/36 = 11/12$

3. An opinion survey of a Sample of 150 adults was conducted. Each person was asked his or her opinion about floating a bond issue to build a community swimming pool. Results are summarized as follows

	Favor	Oppose	Total
Male	60	20	80
Female	40	30	70
Total	100	50	150

Find

a) $P(M) = 80/150$

b) $P(F) = 70/150$

(c) $P(\text{favor}) = \frac{100}{150} = \frac{2}{3}$

(d) $P(M \cap \text{Favor}) = \frac{60}{150} = \frac{2}{5}$

(e) $P(M \cup \text{Favor}) = (80 + 100 - 60)/150 = \frac{120}{150} = 4/5$

Definition: The odds in favor of event E occuring is the ratio

$$\frac{P(E)}{P(E')}$$

provided $P(E') \neq 0$

3. A student believes that the probability of getting an A on the ECO 3401 final exam is 0.1. What are the odds in favor of this occuring?

Let E = gets an A. Then $P(E) = 0.1$ $\quad P(E') = 0.9$.

Odds of an A $= P(E)/P(E') = 0.1/0.9 = 1/9 = 1:9$

Suppose now that the odds of getting an A is 1:5. What is the probability of getting an A

Odds of an A $= P(E)/P(E') = 1/5$

$$= [P(E)/(1-P(E)] = \frac{1}{5}$$

So

$$P(E) = \frac{1}{5}(1-P(E)$$

OR

$$5P(E) = 1 - P(E)$$

$$6P(E) = 1 \quad \Rightarrow \quad P(E) = \frac{1}{6}$$

Homework: pp 421–423, # 1, 3, 7, 13, 17, 21, 27, 31, Conditional Probability

Now we consider probabilities in cases where information about a situation is known in advance.

Example : A die is rolled. What is the probability of getting a 2 given that the outcome is an even number?

$$P(2 \mid \text{even}) = \frac{1}{3} = \frac{\#(2 \cap \text{even})}{\#(\text{even})}$$

In general, if events E and F are associated with an equiprobable sample space and $F \neq \emptyset$, then

$$P(E \mid F) = \frac{\#(E \cap F)}{\#(F)}$$

Examples:

1. An urn contains 2 blue marbles (B_1 and B_2) and two white marbles (W_1 and W_2). If two marbles are randomly drawn without replacement, find the probability that the second marble is white given that the first marble drawn is blue

$$P(W \mid B) = \frac{\#(W \cap B)}{\#(B)}$$

$$\cancel{\#(B)}\ B = \{B_1B_2,\ B_1W_1,\ B_1W_2,\ B_2B_1,\ B_2W_1,\ B_2W_2\}$$

$$(W \cap B) = \{B_1W_1,\ B_1W_2,\ B_2W_1,\ B_2W_2\}$$

So $P(W \mid B) = \frac{4}{6} = \frac{1}{3}$

2. In a survey of 150 people, each person was asked his or her marital status and opinion about floating a bond issue for a community swimming pool. Results below.

	Favor (F)	Oppose (F′)	Total
married (M)	60	20	80
Single (M′)	40	30	70
Total	100	50	150

a. $P(F|M) = \dfrac{\#(F \cap M)}{\#(M)} = \dfrac{60}{80} = \dfrac{3}{4}$

b. $P(M|F) = \dfrac{\#(F \cap M)}{\#(F)} = \dfrac{60}{100} = \dfrac{3}{5}$

Another calculation method. For example,

$$P(F|M) = \frac{\#(F \cap M)}{\#(M)} = \frac{\#(F \cap M)/150}{\#(M)/150} = \frac{P(F \cap M)}{P(M)}$$

3. After the initial production run of a new style of steel desk, a quality control technician found that 40% of the desks had an alignment problem and 10% had both a defective paint job and an alignment problem. If a desk is randomly selected from this run and it has an alignment problem, what is the probability that it has a defective paint job?

$$P(D|A) = \frac{P(D \cap A)}{P(A)} = \frac{0.1}{0.4} = \frac{1}{4}$$

4. If a family has two children, find the probability that both are boys given that one of the children is a boy.

$$P(\text{both} \mid \text{one boy}) = \frac{P(\text{both} \cap \text{one})}{P(\text{one})}$$

$S = \{BB, BG, GB, GG\}$

So, $$\frac{P(\text{both} \cap \text{one})}{P(\text{one})} = \frac{\#(\text{both} \cap \text{one})}{\#(\text{one})} = \frac{1}{3} = \frac{1/4}{3/4} = \frac{P(\text{both} \cap \text{one})}{P(\text{one})}$$

5. A computer hardware company placed an ad for its new modem in a popular computer magazine. The company believes that the ad will be read by 32% of the magazine's readers and that 2% of those who read the ad will buy it. Find the probability that a reader will read the ad and buy the modem.

This time we want to find $P(R \cap B)$. Know that

$$P(B|R) = \frac{P(R \cap B)}{P(R)}$$

So, $$P(R \cap B) = P(B|R)\,P(R)$$
$$= (0.02)(0.32) = 0.0064$$

6. Two cards are drawn without replacement from a standard deck of 52 playing cards. Find the probability that the second card is red.

$$P(R_2) = \underbrace{\tfrac{26}{52}}_{P(R_1)} \cdot \underbrace{\tfrac{25}{51}}_{P(R_2)} + \underbrace{\tfrac{26}{52}}_{P(B_1)} \cdot \underbrace{\tfrac{26}{51}}_{P(R_2)} = \frac{1}{2}\left(\frac{25+26}{51}\right) = \frac{1}{2}$$

So, in this case,

$$P(R_2) = P(R_2|R_1)P(R_1) + P(R_2|B_1)P(B_1)$$

$$= \frac{25}{51}\cdot\frac{26}{52} + \frac{26}{51}\cdot\frac{26}{52} = \frac{1}{2}$$

which is what we had on previous page

7. Two cards are drawn without replacement. What is the probability that both are red

$$P(R_1 \cap R_2) = P(R_2|R_1)P(R_1) = \frac{25}{51}\cdot\frac{26}{52} = \frac{25}{102}$$

8. A company uses one computer chip in assembling each unit of a product. The chips are purchased from suppliers A, B, and C, and are randomly picked for assembling a unit. 20% come from A, 30% come from B, and 50% come from C. 3% of chips from A are defective 4% from B are defective, 1% from C are defective. If one unit is selected at random, what is the probability that it has a defective chip?

$$P(D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C)$$

$$= (0.2)(0.03) + (0.04)(0.3) + (0.01)(0.5) = 0.023$$

9. Urn I contains one black and two red marbles. Urn II contains one pink marble. An urn is selected at random. Then a marble is drawn from it and placed in the other urn. A marble then is drawn from that urn. Find the probability that it is pink

$$P(Pink_2) = P(Pink_2|Urn\ I)\,P(Urn\ I) + P(Pink_2|Urn\ II)\,P(Urn\ II)$$

$$= \frac{1}{4}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = \frac{3}{8}$$

HW: pp. 434-435 1, 7, 9, 11, 29, 33, 47 plus several more