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Historical impulse response of return for Dow Jones averages

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Abstract

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Keywords: Impulse response, stock market history

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Historical Impulse Response of Return for Dow Jones Averages

1. Method

This paper reports a method for deriving an empirical impulse response function of return. The method is applied to rolling 5000 trading day intervals of closing values of three Dow Jones Averages (Industrial -- DJIA, Transportation -- DJTA, and Utility -- DJUA) for the period 5/28/1936 to 1/26/2004, which comprises 17,000 trading days.

For each trading day t, a return, r_t, is computed, which is the change in closing price over the 40 trading days following as a fraction of price, p_t. Next we determine a trading day history window for p_t such that p_t is not included in calculating return values for prices in that history window. Since any return for trading days *t-40* through t includes p_t in the calculation, a 400 trading day history window preceding trading day t that does not include p_t would begin at day $a = t\text{-}400\text{-}40 = t\text{-}440$ and end at day $b = t\text{-}40\text{-}1 = t\text{-}41$. This presentation uses 40 trading days as the return horizon and 400 trading days as the history period throughout.

Hence we have:

t	=	$1,\ldots,17000$ trading days
p_t	=	closing value of Dow Jones Average on trading day t
r_t	=	$(p_{t+40} - p_t) / p_t$ return for trading day t
$R_{a,b}$	=	$\{ r_t \mid a \le t \le b \}$ a set of returns for trading days a to b
$R_{t-440,t-41}$	=	$\{ r_t \mid t\text{-}440 \le t \le t\text{-}41 \}$ Historical Return Set for p_t

A positive impulse is identified if a return, r_t, exceeds the average of the returns in p_t's historical return set $R_{t-440,t-41}$ by a multiple of the standard deviation of the returns in that historical return set. A value of 1.5 is used for this multiplier throughout this paper.

$\check{r}_t = {}^1/_n \sum_{k=1}^{n} r_k$ the average return for the historical return set, $R_{t-440,t-41}$

$s_t = \sqrt{[{}^1/_{n-1} \sum_{k=1}^{n} (r_k - \check{r}_t)^2]}$ the standard deviation for the historical return set, $R_{t-440,t-41}$

$m = 1.5$ a constant multiplier

$$i_t = \begin{cases} 1, & \text{if } r_t > \check{r}_t + m \bullet s_t \\ 0, & \text{otherwise} \end{cases}$$ the impulse for trading day t

Table 1 contains the beginning and ending trading day numbers and the corresponding dates for the intervals used. For each interval the table lists the number of positive impulses which occurred in that interval for each of the three Dow Jones Averages.

Interval	Trading Day		Date		Impulses		
	Begin	End	Begin	End	DJIA	DJTA	DJUA
36-56	1	5000	05/28/36	05/21/56	415	427	395
56-76	5001	10000	05/22/56	05/03/76	376	528	500
60-80	6001	11000	05/11/60	04/17/80	409	522	491
64-84	7001	12000	05/04/64	03/30/84	469	496	481
68-88	8001	13000	04/24/68	03/16/88	541	510	507
72-92	9001	14000	05/16/72	02/28/92	497	466	477
76-96	10001	15000	05/04/76	02/13/96	514	446	441
80-00	11001	16000	04/18/80	01/31/00	525	393	458
84-04	12001	17000	04/02/84	01/26/04	470	378	521

Table 1: Intervals used in terms of trading days and the number of impulses in each interval for each Dow Jones Average.

The method defines the "response" to the impulse in terms of time lags expressed in trading days. This impulse response function is derived from an aggregation of trading days in a response set. Returns for trading days are identified for inclusion in the impulse response return set by relating them to the return for the trading day preceding them by a lag of between 1 and 200 trading days. If a trading day's return is identified as a return associated with an impulse, then the lagged trading day return is a member of the response set. There is an impulse response return set, $I_{a,b,L}$, for each of the 200 lag values for trading day interval from *a* to *b*.

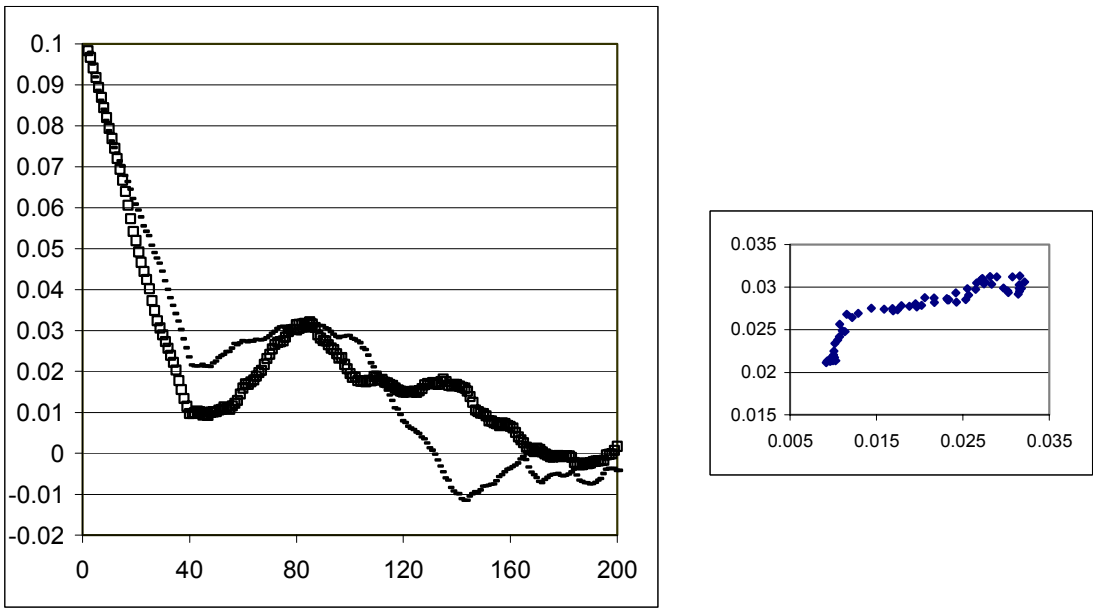
That is, we have:

$L = 1,\ldots, 200$ the lag in trading days

$I_{a,b,L} = \{ r_k \mid i_{k-L} = 1 \text{ and } a \leq k \leq b \}$ the impulse response return set given L from trading day a to trading day b

$\check{\imath}_{a,b,L} = {}^{1}/_{n}\sum_{k=1}^{n} r_k$ where r_k is in $I_{a,b,L}$ the average impulse response return in $I_{a,b,L}$ for lag L

Figure 1(a) shows a graph of impulse responses for the time intervals 36-56 (the 200 $\check{\imath}_{1,5000,L}$ values for the years 1936 to 1956, denoted by the squares in the graph) and 56-76 (the 200 $\check{\imath}_{5001,10000,L}$ values for the years 1956 to 1976, denoted by dashes) for lags of 1 to 200. Figure 1(b) graphs the impulse response values for interval 36-56 on the x-axis and interval 56-76 on the y-axis for lags of 40 to 100. The method is ex-ante for lags of 40 and up, and so it is the impulse responses for those lags that are of interest for forecasting and trading applications.



(a) (b)

Figure 1: (a) Impulse response graphs for intervals 36-56 (squares) and 56-76 (dashes) for lags 1 to 200. (b) Graph of impulse response values for interval 36-56 against impulse response values for interval 56-76 for lags of 40 to 100.

2. Historical comparisons

Table 2 shows correlation coefficient values calculated for each interval, for each of the three Dow Jones Averages, for lags 40 to 100, and for lags 40 to 200. The correlation coefficient is calculated for the impulse response curve for interval 36-56 for the respective Dow Jones Average as correlated against each of the other intervals for the same Dow Jones Average. The correlation coefficients then are:

$c_{X,Y}$ = correlation coefficient as computed from the sets X and Y of values paired as (x_L,y_L) where L is the lag value

C_{a,b,c,d,L_1,L_n} = correlation coefficient as computed from

$$c_{\check{\imath}_{a,b},\check{\imath}_{c,d}}\left(\{\check{\imath}_{a,b,L} \mid L_1 \le L \le L_n\}, \{\check{\imath}_{c,d,L} \mid L_1 \le L \le L_n\}\right)$$

For example, the value in the second column of Table 2 is 0.907. This is the evaluation of $C_{1,5000,5001,10000,40,100}$ for the Dow Jones Industrial Average.

Interval	DJIA		DJTA		DJUA	
	40-100	40-200	40-100	40-200	40-100	40-200
56-76	.907	.734	.434	.627	.654	.385
60-80	.943	.700	.667	.547	.376	.161
64-84	.923	.765	.661	.594	.343	.298
68-88	.910	.805	.700	.642	.334	.269
72-92	.863	.751	.600	.498	.077	.235
76-96	.893	.828	.490	.215	-.125	.124
80-00	-.496	.706	.330	.426	.355	.276
84-04	-.475	.391	.490	.026	.617	.301

Table 2: Correlation coefficient values for the impulse response function of each interval correlated with the first interval 36-56 for each Dow Jones Average.

The correlation coefficient is useful for comparing the impulse response curves in a rough-and-ready way. In Table 2 it may be seen that the correlation coefficient values for lags 40 to 100 change markedly in sign and magnitude for the DJIA for intervals 80-00 and 84-04 as compared with the correlation coefficient values for the intervals which went before.

Table 3 is prepared in a similar way to Table 2 except that the 36-56 interval for the Dow Jones Industrial Average is used as the correlation partner for each of the other intervals (instead of the 36-56 interval in the respective Dow Jones Average.)

Interval	DJTA		DJUA	
	40-100	40-200	40-100	40-200
56-76	.426	.816	.953	.217
60-80	.850	.812	.865	.203
64-84	.875	.827	.865	.395
68-88	.970	.895	.870	.459
72-92	.927	.727	.718	.557
76-96	.803	.537	.600	.683
80-00	.581	.594	.861	.749
84-04	.714	.367	.938	.609

Table 3: Similar to Table 2 but 36-56 interval for DJIA is used as correlation partner for each of the intervals in the DJTA and in the DJUA.

In Table 3 the correlation coefficient values are generally higher for the DJTA and DJUA than they are in Table 2. The correlations continue to be relatively strong through the 80-00 and 84-04 intervals.

Figure 2 shows graphs of the impulse response functions for all of the intervals for the DJIA, except that the curves are offset by the average profit in each of the intervals. Thus, Figure 2 shows excess profit, which is the difference between $\check{\iota}_{a,b,L}$, the average impulse response return in the set $I_{a,b,L}$ for trading days a to b and for lag L, and

the average of returns from trading days a to b, that is the set $R_{a,b}$. A market timing rule which would have worked before 1990 on the DJIA and still worked on the DJTA and DJUA to the end of the study: "Buy at impulse lag day 80 and sell after holding for 40 days".

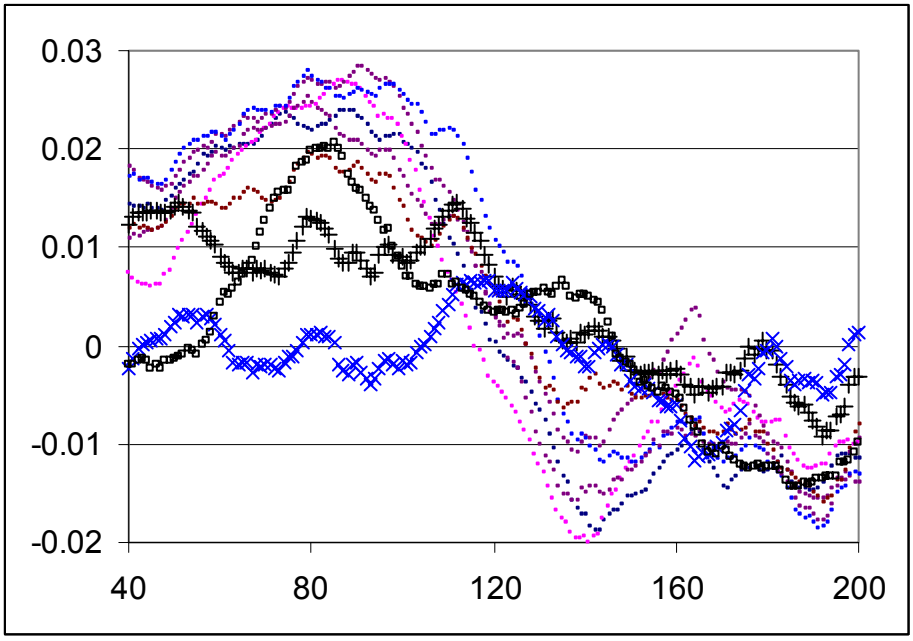
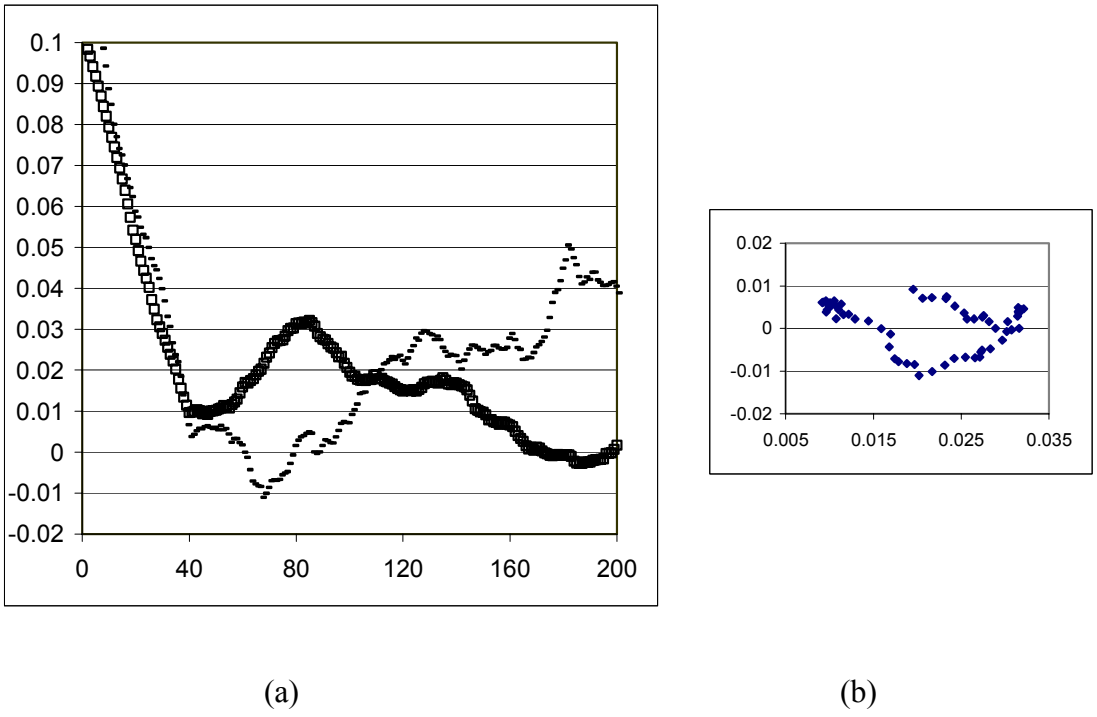


Figure 2: Impulse response for DJIA for each interval, adjusted by average profit in interval. Values for interval 36-56 are denoted by the square, for interval 80-00 by "+", and for interval 84-04 by "x".

The flattening of the impulse response curves for intervals 80-00 and 84-04 (see Figure 2) might be considered to indicate increasing efficiency in the market underlying the DJIA. However, what actually occurred is a transformation of the impulse response curve, which over the 5000 trading day period of interval 84-04 averages out to a flat graph. Figure 3 shows the impulse response for interval 36-56 along with the impulse response for the 2000 day interval from 2/13/1996 to 1/26/2004, which included 162 trading days identified as impulses.



(a) (b)

Figure 3: (a) Impulse response graphs for the 5000 trading day interval 36-56 (squares) and for the 2000 day interval from 2/13/1996 to 1/26/2004 (dashes) for lags 1 to 200. (b) Graph of impulse response values for interval 36-56 against impulse response values for the 2000 day interval from 2/13/1996 to 1/26/2004 for lags of 40 to 100.

3. Conclusion

The stock market acts like an elastic medium which transmits shock waves through time, in this case for price shocks. The stock market information transmission medium was relatively homogeneous from 1936 up until about 1990, when the medium changed drastically for the DJIA, but not for the DJTA and DJUA, though perhaps we can expect the DJTA and DJUA to be transformed in the same way in the future. This work sheds light on the effectiveness of stock charting and may exemplify a general modeling paradigm for aspects of the phenomena referred to as momentum, mean reversion, market efficiency, and over/under-reaction.